UNIVERSIDADE DE LISBOA

FACULDADE DE CIÊNCIAS

DEPARTAMENTO DE ESTATÍSTICA E INVESTIGAÇÃO OPERACIONAL

Optimizing the Master Surgery Schedule in a Private Hospital

Nara Marques Barros

Mestrado em Estatística e Investigação Operacional

Especialização em Investigação Operacional

Dissertação orientada por:
Professora Doutora Inês Marques Proença
Professora Doutora Maria Eugénia Vasconcelos Captivo

2016

I would first like to thank my high school math teacher, Maria Helena Oliveira, for encourage me to apply for an undergraduate program in Applied Mathematics. Without her strong presence during these three years of my life, this thesis could never have existed.

Secondly I would like to thank my advisers, Professor Inês Marques Proença and Professor Maria Eugénia Vaconcelos Captivo, for all the patient, support and inspiration. During my thesis execution, they share without any doubt a lot more than knowledge in the field of Operation Research.

I would also like to leave a special thank you to the head doctor of the surgical suite of the hospital under study, and to the surgical scheduling department nurse who were always available to explain and clarify me about the hospital processes, the medical practices, and its implications.

Last, but not least, I would like to thank my parents for supporting and prioritizing my education, and my boyfriend for always asking: ”How is it going?”, and for hearken all my frustrations and accomplishments.

Nara

Abstract

This dissertation presents an automated method for creating a cyclic master surgery schedule (with a week horizon), and describes the reality and the results of a case study applied in a medium-sized Portuguese private hospital.

Four objectives are taken into account when building the master surgery schedule, which can have OR time allocated to a specific surgeon, or to a surgical specialty. Firstly, the resulting workload at the hospitalization units should be leveled as much as possible. Secondly, the operating rooms are best allocated if shared as little as possible between different surgical specialties (i.e., by being shared by surgeons of the same surgical specialty). Thirdly, the surgical specialties are best allocated when the highest number of surgeons not already assigned that belong to the surgical specialty are available. Lastly, the weekly OR time assigned to surgeons or surgical specialties must be as close as possible to the corresponding OR time used in the last trimester (so the master surgery schedule is renewed based on the recent demand for surgeries).

The surgery duration is not assumed to be deterministic. Since the duration of the surgeries are highly dependent on the type of surgery, and dependent on the surgeon and on the surgical specialty of the surgeon performing the surgery, Sturges' rule is considered in order to incorporate the empirical distribution of the stochastic variable in the model.

Besides the constraints related with the objectives, the developed model incorporates structural constraints which ensure that surgeons and surgical specialties share the OR time properly, such as, capacity constraints that limit the available blocks on each day, and others related with hospital requirements. The number of required OR time blocks per surgical specialty is not given as input as it usually happens on other studies available in the literature.

The method relies on mixed-integer linear programming techniques involving the solution of multiobjective optimization problems. Since the problem' objective function is formulated as a weighted sum of the multiple criteria presented, the model does not provide an overall solution, and after different algorithm runs, it is up to the decision maker to choose the best solution.

Keywords: Health care, Operating rooms, Master surgery schedule, Mixed-integer linear programming, Multiobjective approach

Resumo

O sistema de saúde e os fatores que determinam a sua evolução são de grande complexidade. O desenvolvimento científico, tecnológico, social e económico a que temos assistido nas últimas décadas possibilitou a resolução de muitos dos problemas relacionados com os cuidados de saúde que enfrentávamos, mas também contribuiu para a descoberta de novos problemas mais complexos. Entre outros, as alterações nas necessidades dos cuidados de saúde motivadas pelo aumento da esperança média de vida, o progressivo envelhecimento da população, e o aumento da incidência e prevalência de doenças crónicas, deu origem aos novos desafios que o sector dos cuidados de saúde enfrenta atualmente.

O sector dos cuidados de saúde em Portugal sofreu grandes alterações não só devido ao envelhecimento da população, mas também devido à crise financeira portuguesa (que se desenvolveu em consequência da crise da dívida pública da Zona Euro). A crise financeira portuguesa levou ao corte de orçamentos nos hospitais públicos e também no setor privado.

A gestão do fornecimento de serviços de saúde está, pois, a tornar-se cada vez mais importante e exigente. Uma unidade hospitalar que é de particular interesse é o bloco operatório. Dado que uma grande percentagem das admissões hospitalares é devida à necessidade de intervenção cirúrgica (Guerriero & Guido 2011), o bloco operatório é um recurso hospital que representa um elevado nível de custo e receita, o que deixa pouco espaço para uma gestão ineficiente ou comunicação deficiente.

O planeamento e agendamento do bloco operatório pode ser encarado como um processo de otimização hierarquizado em três fases: *case mix planning* (ou nível de decisão estratégica, em que são tomadas as decisões relacionadas com a oferta cirúrgica do hospital), *master surgery planning* (ou nível tático, em que é definido um horário cíclico que divide, entre cirurgiões ou especialidades cirúrgicas, o tempo de funcionamento das salas de operação), e *surgery scheduling* (ou nível operacional, em que os procedimentos cirúrgicos são marcados para uma sala operatória e dia específico), tal como referido no trabalho de Marques et al. (2012).

Este trabalho apresenta e estuda a realidade de um hospital privado de dimensão médio localizado em Lisboa. O hospital tem oito salas de operação idênticas, estando apenas sete completamente equipadas e operacionais e nenhuma delas dedicada a um serviço específico, treze valências ou especialidades cirúrgicas, cerca de 224 cirurgiões ativos. Mais de 8400 cirurgias realizadas no ano de 2014. O hospital tem serviço de emergência, mas dado que a percentagem destes procedimentos é muito reduzida face ao volume global de cirurgias, este serviço não faz parte do estudo realizado. Em termos de cirurgias programadas, o hospital realiza cirurgias de ambulatório e convencionais. Pacientes sujeitos a uma cirurgia de ambulatório têm admissão e alta em menos de vinte e quatro horas. Em oposição, pacientes a que seja realizada uma cirurgia

convencional terão que recuperar pelo menos uma noite numa das unidades de hospitalização (internamento ou cuidados intensivos). Segundo a análise efetuada aos dados das cirurgias realizadas no hospital ao longo de 2013 e 2014, e segundo a administração do bloco operatório, o volume de trabalho do mesmo tem vindo e continua a aumentar, incrementando a dificuldade de o gerir de forma eficiente. Esta realidade hospitalar difere significativamente da realidade dos hospitais públicos por não existir lista de espera para cirurgia, e por, em grande maioria, ser o cirurgião a propor ao paciente, no momento da consulta, a data para o procedimento cirúrgico.

Este estudo tem como objetivo criar um *master surgery schedule* automatizado, de acordo com os requisitos específicos do hospital, por forma a aumentar a eficiência do bloco operatório. Este trabalho insere-se no segundo nível de otimização. No entanto, não pode ser considerado um problema exclusivamente inserido neste nível de decisão, pois não considera o tempo a ser atribuído a cada cirurgião ou especialidade cirúrgica como um *input* do modelo. A decisão inerente a este valor é considerada como pertencente ao primeiro nível de decisão. Calculando este valor no processo de otimização do segundo nível, este trabalho difere significativamente do de Beliën et al. (2009), apesar de este ser o trabalho mais próximo de que temos conhecimento em termos de abordagem ao problema e definição de variáveis. As especificações do problema diferem também do trabalho mencionado, dado que cada hospital tem uma realidade hospitalar particular, apesar de os desafios diários, preocupações e dificuldades de implementação serem, genericamente, idênticos. À semelhança do estudo referido, este trabalho considera que a duração das cirurgias não é determinística. É usada a regra de Sturges, por forma a incorporar, no modelo, a distribuição empírica da duração das cirurgias. A duração das cirurgias é conhecida como sendo estritamente dependente do tipo de procedimento cirúrgico, consequentemente dependente da especialidade cirúrgica, e dependente do cirurgião que executa o procedimento (nomeadamente, por ser mais ou menos experiente na realização da cirurgia). Com o uso desta abordagem, espera-se simplificar a tarefa do diretor do bloco operatório, reduzir o conflito entre cirurgiões e o responsável pelo planeamento (sem perda de confiança por parte dos cirurgiões), e aumentar a eficiência e robustez do bloco operatório, reduzindo a variabilidade na sua produção, e consequentemente reduzindo a variabilidade da procura pelos serviços subsequentes (como as unidades de hospitalização). Pretende-se ainda demonstrar, nomeadamente junto do hospital, o potencial das técnicas de programação linear inteira mista na criação de boas sugestões de mudança neste âmbito de atuação.

De forma mais precisa, este trabalho tem o intuito de alocar cirurgiões e especialidades cirúrgicas ao tempo de bloco de operatório disponível, considerando restrições estruturais gerais (p. ex. de capacidade) e restrições particulares provenientes dos processos implementados no hospital. Consideram-se quatro objetivos: reduzir a variabilidade da procura pelas unidades de hospitalização, nivelando o máximo possível, ao longo dos dias, a carga de trabalho resultante do centro cirúrgico; concentrar cirurgiões da mesma especialidade cirúrgica o máximo possível na mesma sala operatória, reduzindo o tempo de intervalo entre cirurgias; alocar tempo de bloco às especialidades cirúrgicas quando o maior número de cirurgiões não alocados estejam disponíveis; e renovar o horário cíclico, com uma semana de horizonte temporal, tendo por base o histórico mais recente de cirurgias. A metodologia criada recai, portanto, num modelo multiobjetivo, formulado como a soma ponderada dos quatro critérios descritos. Ao correr o algoritmo para diferentes pesos dos critérios, tem-se uma visão sobre o espaço das soluções, não se obtendo uma solução ótima única global. Compete, portanto, ao diretor do bloco decidir sobre qual a ponderação dos critérios, e consequentemente a solução, que aplicada à sua realidade particular, é considerada a melhor. Nesta fase, são avaliados vários *trade-offs* (dada a natureza conflituosa dos critérios da função objetivo ponderada).

Foram recolhidos, junto do hospital, os dados das cirurgias realizadas ao longo dos anos de 2013 e 2014. Depois de analisados os dados, e ter sido possível extrair algumas métricas como p. ex. a taxa de ocupação real do bloco e a variabilidade do volume de cirurgias ao longo da semana e do ano, foi possível criar três instâncias de teste. Foram também recolhidos três *master surgery schedules* que estiveram em vigor durante o período de tempo de janeiro de 2013 a março de 2015. Foi, por isso, possível comparar os resultados obtidos pelo modelo e o plano real em vigor. Concluiu-se que o modelo produz soluções de qualidade, de acordo com as especificações e necessidades do hospital. Algumas das soluções encontradas obtêm melhor resultado para as várias métricas apresentadas (critérios da função objetivo) do que a solução implementada no hospital, o que permite estabelecer que o modelo desenvolvido gera *master surgery schedules* que melhoram o desempenho do bloco operatório, tal como proposto inicialmente.

Palavras-chave: Cuidados de saúde, Bloco operatório, Plano mestre de cirurgias, Programação linear inteira mista, Abordagem multiobjetivo

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The healthcare sector and the factors that determine its evolution are highly complex. The scientific, technological, social and economic development that has been witnessed in recent decades allowed to solve many of the past health problems, but it has also contributed to discover newer and more complex problems. Among others, the changes in the needs of healthcare motivated by the increase of the life expectancy, progressive aging of the population, and higher incidence and prevalence of chronic diseases, gave rise to the new challenges that healthcare sector is facing today.

It is known that the Portuguese healthcare sector has suffered major changes not only due to population aging, but also due to the Portuguese financial crisis (from 2010 to 2013), which developed in the context of the Eurozone public debt crisis. The Portuguese financial crisis led to budget reduction not only in Portuguese public hospitals, but also in the private sector.

Given that, the management of healthcare services is becoming more and more challenging, and one unit of special interest is the surgical suite (Cardoen et al. 2010). Since a high percentage of hospital admissions is due to surgical interventions, surgical suites drive a high level of cost and revenue as a hospital resource (Guerriero & Guido 2011), leaving little room for inefficient management or poor communications and visibility.

Operating room (OR) planning and scheduling is known as being a three-stage hierarchical optimization process, as seen in the work of Mannino et al. (2012), Marques et al. (2012), Marchesi & Pacheco (2016), and in the literature reviews of Cardoen et al. (2010), and Guerriero & Guido (2011), among others.

In the first stage, usually called case mix planning or strategic decision level, decisions concerning the hospital's supply for surgery are made. Since planning the case mix is a long term decision, it is usually conducted on an annual basis along with the construction of hospital budget (which includes the number and types of surgical procedures to be performed, the medical staff available, and relevant costs and targets) (Marques et al. 2012). At this stage, and taking into account the expected demand for each treatment type and the resources available, an operational research problem arises. Following the administration perspective, it aims to maximize the revenue or minimize the costs, while optimizing the distribution of service between available resources (e.g. ORs, surgical specialties, specialized staff, and equipment) (Guerriero & Guido 2011). Blake & Carter (2002) presented a linear goal programming approach for allocating resources in hospitals. The two developed models permit to investigate the trade-offs between financial return (as funding) and resource allocation. This problem is also tackled by Kuo et al. (2003) who created a linear programming routine to determine the optimal mix of surgical OR time allocation in order to maximize the income. Testi et al. (2007) have studied this problem in a three-stage hierarchical approach for OR planning and scheduling. For this stage, the authors used a bin packing-like model in order to determine the number of sessions to be weekly scheduled for each ward.

Master surgery planning or tactical decision level is the second stage of OR planning and scheduling. It involves defining a cyclic timetable in which is established the ORs available, their opening hours, and how both (rooms and time periods) are divided among surgeons (also called surgical teams) or surgical specialties. In a master surgery schedule (MSS) the allocation of surgeons or surgical specialties to an OR time period (also called block), give the priority of usage of the corresponding room. Whenever the OR opening hours or the number of available rooms change, a new MSS must be defined in order to achieve a good usage of the resources inside and outside the OR. This changes can occur as a result of the changes on the previous stage (e.g. annual variations in funding), or as a response to the demand seasonal fluctuations (e.g. summer and Christmas holidays). Therefore, operational researchers may produce various MSS per year (Blake et al. 2002, Beliën & Demeulemeester 2007, Marques et al. 2012).

Some hospitals choose to completely allocate each room to a surgical specialty, where others choose to share the allocation of a room with different surgical specialties, or even specific surgeons. The former occurs mainly because of the highly setup costs and time consuming activities for preparing the rooms for some surgical specialties. This type of relevant information must be taken into account in the construction of a MSS. Other relevant information is resources availability, such as surgeons, specialized staff, and medical equipment (extra to the room basic equipment), since their unavailability can cause delays, or other performance issues (Gomes 2014).

This stage of OR planning and scheduling has a greater range of approaches. Blake et al. (2002), Vissers et al. (2005), Testi et al. (2007), and Mannino et al. (2012) used exact methods, while others used heuristic approaches (Blake & Donald 2002, Beliën & Demeulemeester 2007, Beliën et al. 2009, van Essen et al. 2014, Hosseini & Taaffe 2015, Marchesi & Pacheco 2016), or even simulation models (Cappanera et al. 2014).

Blake et al. (2002) described an automated methodology to create a MSS, and presented a hospital's experience of developing a consistent schedule that minimizes the shortfall between each surgical group's target and the corresponding actual assigned OR time, using integer programming techniques. The authors made clear that their model is appropriate for hospitals with limited OR budgets, in opposition to hospitals at which OR time is always provided in order to generate additional revenue using surgical resources. The authors concluded that their methodology simplified the OR manager's task, and believed that the conflict among surgeons and between surgeons and the OR manager was reduced due to the schedule being produced in a consistent, and unbiased manner. Vissers et al. (2005) solved a mixed-integer linear programming model, which is used to evaluate scenarios for the second as well as the third stages of OR planning and scheduling. The authors also demonstrated the potential of integer programming for providing recommendations for change. Testi et al. (2007) solved the problem of maximizing surgeon preference on their three-stage hierarchical approach. Mannino et al. (2012) created two major variants: the first asks for leveling patient queue lengths among different specialties, while the second aims to minimize the overtime. Firstly, the authors created a mixed-integer linear formulation, and then, since the estimation of demand levels is affected by uncertainty, they developed a light robustness approach to the latter variant.

Blake & Donald (2002) presented an integer linear programming approach for the problem of equitably allocating OR time to surgical departments, and a post-solution heuristic, which has also greatly reduced the conflict between surgeons and the OR planning manager. Beliën & Demeulemeester (2007) established models for building MSS with leveled resulting bed occupancy, and developed mixed-integer programming based heuristics and a metaheuristic (simulated annealing) to solve the models. It was the first published work that specifically presented optimization models which aim to level the resulting bed occupancy, and enabled to predict performance measures as the daily expected bed occupancy, the corresponding variance, the expected bed shortage, and the corresponding daily bed shortage probability. The authors considered both the number of operated patients per block, and the length of stay of each operated patient to be dependent on the type of surgery, therefore being considered stochastic variables. Note that, being an exclusively second stage problem, it receives as input the number of required OR time blocks per surgeon. Beliën et al. (2009) presented a decision support system for creating a cyclic MSS, based on the previous described paper. The authors considered three objectives when building the MSS: the resulting bed occupancy at the hospitalization units should be leveled as much as possible, operating rooms should be shared as little as possible between different surgeon groups, and the MSS is preferred to be as simple and repetitive (from week to week) as possible. The authors formulated a multiobjective mixed-integer linear programming model, a multiobjective quadratic optimization problem, and developed a simulated annealing metaheuristic. The developed software does not provide an overall best solution, since different algorithm runs lead to different solutions, and it is up to the decision maker, after visualizing different schedules, to choose the one he prefers. While Beliën & Demeulemeester (2007) focused on minimizing the total expected bed shortage, van Essen et al. (2014) developed two approaches to reduce the number of required beds: a heuristic approach based on local search, and a simplified approach that reduces the complexity of the problem, and makes it possible to solve the resulting problem as an integer linear programming model. The authors' approach considers the expected number of emergency patients, despite not considering the stochastic nature of the arrival process of emergency patients. Hosseini & Taaffe (2015) proposed an algorithm that allocates OR time blocks based on demand variability, considering for both over-utilized and under-utilized time, not in a capacity-constraint situation. The authors also studied the effect of turnover time on the number of ORs that need to be assigned. This study enhances that block adjustments can be even harder than assigning OR time to new surgeons, since there is always competition between surgeons for the OR time, and its reduction can cause loss of trust, when there is no quantifiable data to justify the decision. Marchesi & Pacheco (2016) developed a model for creating a MSS, when the hospital's goal is to minimize the difference between the OR time assigned to each surgical specialty and its demand, and the unmet demand. The authors also successfully tested the efficiency of a genetic algorithm in this context.

More studies were found in the literature considering and trying to overcome the uncertainty in surgeries' duration (van Oostrum et al. 2008), and in patients' length of stay on recovery units (Vanberkel et al. 2011). van Oostrum et al. (2008) proposed a mathematical program containing probabilistic constraints to deal with stochastic nature of the duration of surgical procedures, and the unbalanced scheduling of the surgical theatre, which often causes demand fluctuations in other departments of the hospital, such as recovery units. Planned slacks are used in order to overcome the uncertain duration of surgical procedures. The authors solved the problem using a column generation approach, in a two-phase decomposition method, maximizing the OR utilization, and leveling the demand for wards and intensive care units. Vanberkel et al. (2011) stated that post-surgical activities are quite sensitive to the activities in the OR, and so find it important to define the workload of downstream departments as a function of the MSS. The authors developed a model that computes the ward occupancy distribution, the patient admission and discharge distributions, and the distributions for ongoing interventions, and then supports the development of a MSS. This way department managers can determine their workload by aggregating tasks associated with recovering patients after being operated. Cappanera et al. (2014) were focused on both sources of variability. The authors considered that their study help hospital managers to understand the advantages and disadvantages associated with three scheduling policies in different operational circumstances. They developed a mixed-integer programming model that considers three performance criteria: the efficiency related to scheduling a large number of surgeries, the fair balancing of the workload between the surgical resources, and the robustness to surgical time and length of stay variability. All the policies maximize the first criterion, but each scheduling policy corresponds to a different balancing criterion. The authors used a discrete event simulation model to assess the robustness of the schedules produced by the MILP model. The simulation model samples surgical times and length of stay from both empirical distributions and theoretical (lognormal) distributions. Fügener et al. (2014) were also interested on the downstream costs resulting from the MSS. The authors stated a stochastic analytical approach, that calculates for a given cyclical MSS the exact demand distribution of patients both in the ICU and the wards, and presented measures (as fixed capacities and staffing levels) to estimate the downstream costs resulting from the MSS. They proposed exact and heuristic algorithms (branch-and-bound and simulated annealing) to minimize these costs.

A different approach comes from Beliën et al. (2006), who developed a software system that instantaneously visualizes the impact of the MSS on the consumption patterns for multiple resources throughout the hospital. Since the authors agreed that the MSS can be seen as the engine that drives the hospital, they considered that decision makers must have a clear vision on how the demand for resources is related to the MSS. The system is considered to be very promising for helping the development of the MSS, and for improving the efficiency of resource utilization. However, the authors claimed their system to be deterministic and simple, which can be seen as a flaw: it can predict accurately the load of resources with deterministic utilization (e.g. use of equipment), in opposition to resources where the utilization is subject to high uncertainty (e.g. bed occupancy). Van Oostrum et al. (2010) also summarized the advantages of using a MSS approach: it provides the required medical autonomy of surgeons, while it obtains an OR high efficiency and robustness due to the previously alignment of resources inside and outside the OR. A MSS approach aims to minimize the week-to-week variation in OR production, and consequently the resulting demand for other hospital resources (due to its cyclic nature), which improves clarity and predictability of work processes, levels workflow, and optimizes patient flows. Besides several implementation issues from an advanced planning approach (such as MSS), as the availability of reliable data and weak cooperation between different actors in the hospital organization, the authors recommended all hospitals to consider the implementation of a MSS approach. The authors addressed the typical implementation issues, and offer guidelines for dealing with them.

Note that, at this stage, no surgeries are scheduled. However, medical staff can visualize when and where surgeries can be performed. It is an important stage of operating theater optimization, since it establishes the mix of surgical procedures taking place in the near future. Yet some hospitals use an open scheduling strategy (also called first-come, first-served strategy), in opposition to a block scheduling strategy (used when considering a MSS). An OR planning and scheduling problem using an open scheduling strategy is presented e.g. in the work of Fei et al. (2010).

Surgery scheduling or operational decision level is the last stage of the three-stage process being defined, and it can be divided into advance scheduling and allocation scheduling. Advance scheduling consists of scheduling each elective surgery for a specific room and day. The surgical team must also be defined at this stage. Each surgery can be assigned to a specific period of the day, or a set of surgeries can simply be assigned to a day and then, ordered, which is called allocation scheduling. Since the short-term nature of this decision level, and since it involves each specific patient, this stage must be performed in a daily or weekly basis. The work of Hans et al. (2008) is only focused on advance scheduling, while the work of Cardoen et al. (2009) is only focused on allocation scheduling. Some authors considered elective case scheduling in a single problem (Marques et al. 2014), while others studied both advance and allocation scheduling in two separate problems (Fei et al. 2010). A different approach is presented by Testi et al. (2007), who, after determining the optimal MSS, scheduled patients are selected based on the developed discrete-event simulation model.

The present case study entails a private hospital in Lisbon, and intends to create an automated MSS according to the specific requirements of the hospital. The main aim of this study is to propose ways to enhance the efficiency of the hospital's surgical suite. Despite of the great range of approaches that are available in the literature, concerning our knowledge, no study has been carried out that fully considers the reality of the hospital under study. Although the general reality of hospitals, its daily challenges, concerns and implementation difficulties are similar, each hospital appear to have their particularity, which in most cases induces the department manager or operational researcher to start his own formulation, based on the most related work published. Notwithstanding, it is expected that this work findings are in accordance with the authors above: the usage of a MSS approach simplifies the OR manager's task, reduces the conflict between surgeons and the planning manager (without surgeons' loss of trust), increases the OR efficiency and robustness, reduces the intra-week and week-to-week variability in OR production, and consequently reduces the variability on the resulting demand for downstream units (e.g. recovery units), and in the end demonstrates the potential of mixed-integer programming techniques for providing great recommendations for change.

Bearing in mind the definition of the OR planning and scheduling as a three-stage hierarchical optimizing process, this work falls within the second stage, master surgery planning. However, and unlikely Beliën & Demeulemeester (2007), the number of required OR time per surgeon or surgical specialty is not given as input, and so it must be calculated, not being considered an exclusively second stage problem. The presented problem differs significantly from the above since it implicitly has behind a huge question of the case mix planning stage.

Bearing in mind the approach to the problem plus the use of the same type of variables in the developed model, the closest work is that of Beliën et al. (2009). Notwithstanding, and as remarked before, the specifications of this problem are different from those observed in the case study mentioned above. More precisely, this study aims to: (1) assign surgeons and surgical specialties to the available OR time, while reducing the variability at the care units, in order to level the workload as much as possible, (2) concentrate surgeons from the same surgical specialty as much as possible in the same room, in order to reduce surgeries' turnover time, (3) allocate an OR time with the highest number of available and not individually assigned surgeons to the corresponding surgical specialty, and (4) renew the MSS based on the recent historical data. A multiobjective mixed-integer linear programing model is developed. Since the problem' objective function is formulated as a weighted sum of the multiple presented criterion, the model does not provide an overall solution, and after different algorithm runs, it is up to the head doctor of the surgical suite to choose the best solution.

This work proceeds in Chapter 2 with a description of the problem, and some hospital specifications. Chapter 3 presents a mixed-integer linear programming model for the MSS approach, followed by a discussion of the complexity of the problem, the description of the model implementation process and of the real instances in Chapter 4. Chapter 5 presents the results of the computational experiments performed using the hospital data, and the analysis of the solutions. Finally, conclusions are reported in Chapter 6.

Chapter 2

Problem description and case study

The case study entails a private hospital located in Lisbon, which is part of a reference group in the health care sector in Portugal. The group is known for the high quality clinical services in the medical and surgical area. The group has five hospitals and four clinics spread from north to south of Portugal and relies on the work of more than 4500 professionals.

The hospital under study offers a wide range of health care services as specialty consultations, complementary diagnosis exams, emergency care, surgery, maternity, medically assisted procreation and inpatient units. The maternity has two inpatient units, obstetrics-gynecology inpatient unit and neonatal intensive care unit, to take good care of mothers and babies. The hospital had its services all concentrated in one building. In 2015, due to huge operational growth, the hospital moved some services to a new building (namely, specialty consultations and complementary diagnosis exams). The remaining services had the opportunity to expand in the old building. This study focus on the surgical suite and the surgical recovery units (including wards), which are separated from the above-mentioned units and the center for medically assisted procreation.

The hospital performed nearly 7800 surgeries in 2013 and more than 8400 in 2014. It is expected that the number of surgeries per year continues to increase. In Figure 2.1 is shown the evolution of the number of surgeries performed per week from the first week of 2013 – week 1 – to the last week of 2014 – week 105. Note that the low peaks occur at the beginning and end of the year - weeks 1, 52, 53, 104 and 105. These weeks coincide with the Christmas and New Year holidays.

The hospital has emergency service, but the problem is exclusively dedicated to elective surgeries, since non-elective surgeries represent only two percent of all surgical procedures. An elective surgery is a non-emergency surgical procedure that can be either conventional or ambulatory. These two types of surgeries are alternatively called inpatient and outpatient

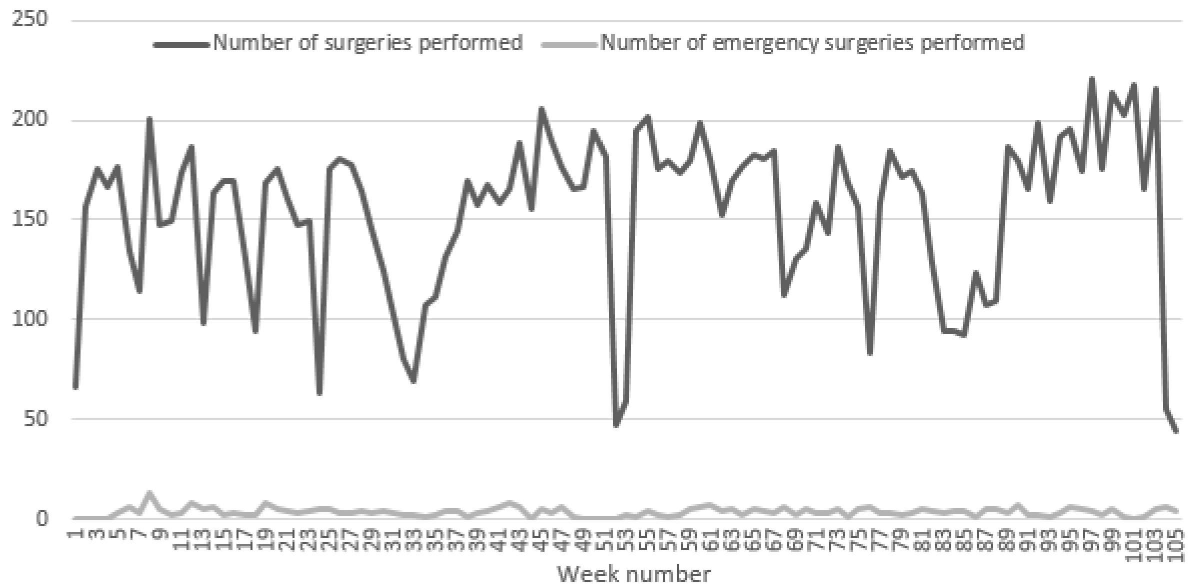

Figure 2.1: Evolution of the number of surgeries performed per week (2013 and 2014)

surgeries, respectively. Inpatient surgeries, as opposed to outpatient surgeries, require at least one overnight hospital stay, namely hospitalization. For outpatient surgical procedures the entry and departure of the patient in the hospital occur in less than twenty-four hours. The inpatient and outpatient flows will be described later. Figure 2.1 also shows the evolution of the number of emergency surgeries performed per week from the first week of 2013 to the last week of 2014, and helps to realize the irrelevance of the number of emergency surgeries in the number of surgeries performed.

The hospital has one central surgical suite with eight identical operating rooms (ORs), but one of these ORs is still closed. The unused OR is ready to be open, although it is not equipped, since until now the workload does not justify the initial cost concerning the equipment and the fixed costs of keeping it open. There is no dedicated ORs, so all the seven available ORs can be used to perform all types of surgery. Additional or specialized equipment has to be required when needed, so it can be prepared and moved in time. The surgical suite is open between 8am and 11pm, from Monday to Friday. Each day is divided in two shifts: Morning (from 8am to 4pm) and Afternoon (from 4pm and 11pm). The surgery schedule has to respect this opening hours and so no surgery should be planned using extra time, despite nearly four percent of surgeries have been performed outside this opening hours in the period under study (2013 and 2014).

Thirteen surgical specialties and more than 200 active surgeons compete for the OR time (in each trimester). Figure 2.2 shows the total number of active surgeons and the average number of

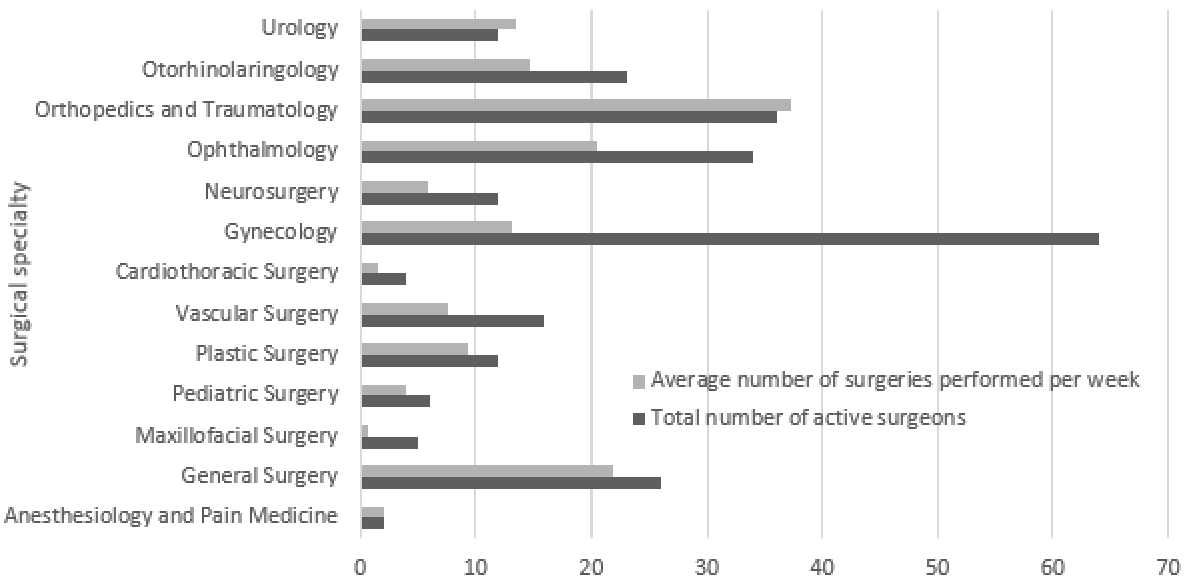

Figure 2.2: Number of active surgeons and average number of surgeries performed per week, per surgical specialty (2013 and 2014)

surgeries performed per week, per surgical specialty (in 2013 and 2014). As shown in this figure, a surgical specialty with a larger number of surgeons has not necessarily performed a higher number of surgeries, which leads us to believe that some specialties have greater interchange of active surgeons on the OR per week. In Figure 2.3 is shown the number of surgeries performed and of OR hours used by each surgeon in the period under study. This figure helps to understand that the number of patients operated in a shift (with the same capacity) varies considerably with the assigned surgeon. Some surgeons, unlike the majority, do not need much time to perform many surgeries, and so can perform a larger number of surgeries and send a greater number of patients to the hospitalization units.

In Portugal, it is usual for surgeons to operate at any time of the day within the opening hours of the surgical suite, if the allocated specialty of an available room matches the surgery's specialty (Marques et al. 2014). Although it is considered a block-scheduling system, it has the simplicity and flexibility of an open scheduling strategy. This hospital also operates in a block-scheduling system, which is the most widely used (Guerriero & Guido 2011). In fact, the practice of this hospital is to assign rooms, days and shifts to surgeons or surgical specialties, creating a more elaborated master surgery schedule (MSS) than the commonly used in Portugal. The head doctor of the surgical suite meets with the most important doctors (the ones with more surgical workflow) in his office when needed (usually, every three months). The head doctor tries to discuss the availability of the surgeons and to negotiate the amount of weekly hours requested

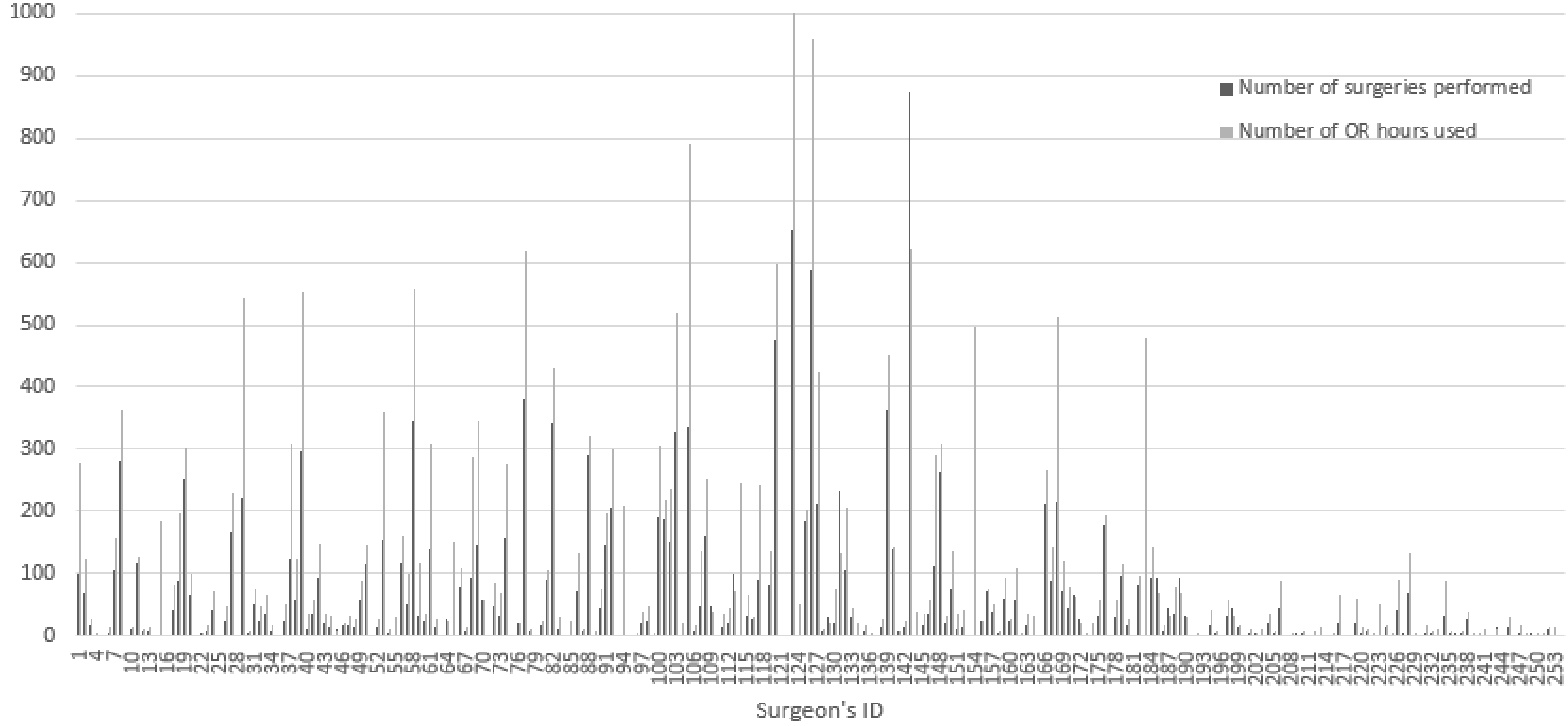

Figure 2.3: Number of surgeries performed and of OR hours used per surgeon (2013 and 2014)

by each surgeon. Then, without any optimization procedure, he creates a new MSS that is sent to the surgical scheduling department and displayed at the entrance of the surgical suite. The task of building a cyclic MSS, with a weekly horizon, is becoming even harder, since the demand for surgeries has been increasing, as well as the number of active surgeons to manage, and the fact that surgeons' estimations for their weekly required time are in most cases inaccurate.

During the patient consultation with the doctor, the surgical procedures are discussed and they try to arrange a date that best fits both of them, considering the MSS, personal preferences, availability, and common practices. Patient's travel time and patient's age are other examples of aspects taken into account. An attempt is made not to schedule patients with considerable travel distance between his residence and the hospital in the begging of the day. Children up to five years old, opposing to adults, cannot easily stay sober when the surgery is performed and the absence of food can cause exasperation on the hospital's staff, other patients and companions, so it is preferable to schedule these surgeries as early as possible in the morning. The nurses or the administrative assistant in the surgical scheduling department receive the surgical proposal from the doctors and confirm or reschedule the surgery based on ORs' availability, also using the MSS as a guideline. It is possible that some type of "negotiation" between administrative assistants and doctors, and also between doctors and patients, exists. The administrative assistants also perform some changes, after confirming all the surgeries of a certain day, in order to make the schedule more consistent. For example, if a room only has allocated surgeries from 8am to 11am in the morning and from 4pm to 10pm in the afternoon, while another room just has allocated surgeries from 11am to 3pm, if possible the administrative assistant will consolidate all the surgeries in the same room. This consolidation leads to a reduction of costs, since the costs of the unallocated room decrease. Sometimes, this consolidation is not possible given the nature of the surgeries of the surgical specialties. For example, "dirty" surgeries can only be consolidated with surgeries from the same type, as well as "clean" surgeries can only be consolidated with "clean" surgeries. The administrative assistants also try to consolidate surgeries of the same surgical specialty in the same room. For example, the surgical specialty gynecology performs surgeries that require the surgical imaging equipment. If the surgeons of this surgical specialty are concentrated in the same room, the surgical imaging equipment will not have to be moved, which is considered a good practice. Notice that these tasks consume a mass portion of the administrative assistant's time, since doctors deliver the surgical proposal on a paper and the administrative assistants have to copy it to the information technology (IT) platform, although surgeons can directly introduce the information of the surgical proposal on the system. Also notice that each patient is previously assigned to a surgeon (at the specialty consultation) and thus, when planning, patients and surgeons are already paired.

After a surgery is performed cleaning and disinfecting protocols should be carried out. The cleaning and disinfecting protocols average duration is fifteen minutes. A “clean” surgery (e.g. a cataract surgery - phacoemulsification) requires a cleaning time of about ten minutes, while a “dirty” surgery (e.g. a resection of the colon - hemicolectomy) requires a cleaning time of about twenty minutes. In exceptional cases, the surgery is highly contagious (as, for instance, when the patient has HIV infection), and requires a cleaning time of about thirty minutes. However, this type of surgeries is allocated lastly (i.e. they are usually scheduled to the end of the day, when no other surgery is expected to take place afterwards). The department staff is partly composed of nurses, who help in the training of administrative assistants, which end up getting sensibility for these situations, and thus taking them into account when managing the allocation of surgeries.

Usually each room has an allocated anesthetist. The shift’s duration of the ORs was defined taking into account the exchange time of the operational anesthesia care team. There is a team that operates in the first shift of the day, and another one that operates in the second shift of the day. These teams are not fixed throughout the week. Sometimes due to the low expected flow, it is used a volant anesthetist. An anesthetist is called volant when he is allocated to more than one room in the same shift. However, the allocation of an anesthetist to more than one room in the same shift can only occur in specific circumstances. The surgeries of the two rooms cannot be carried out simultaneously. For example, if a room only has allocated surgeries from 8am to 9:30am in the morning and from 2pm to 4pm in the afternoon, while another room just has allocated surgeries from 10am to 1pm, and it was not possible to consolidate all the surgeries in the same room given the nature of the surgeries of the surgical specialties, then a volant anesthetist can be allocated to both rooms.

In the hospital under study, it is known from practical experience that the occupation rate of other units (namely beds in the recovery units such as the wards) and the shortage of resources (such as staff - nurses and nursing aides - specialized equipment and clinical materials) do not limit the activity of the surgical suite. However, this was not always true, and that’s why the hospital was expanded in 2015. Now it has a new building for consultations and exams. The wards were increased in the older building, and consequently the number of beds available. It is important to level as much as possible the workload in the OR and consequently in care units, in order to better manage and allocate the staff in rotating shifts. Thus, we intend to avoid last minute changes in the operational schedules as unplanned extra-hours, and too exhausting shifts due to the workload being too high for the number of nurses and nursing aids assigned.

As shown in Figure 2.4 (in schematic form), inpatient surgeries scheduled to the first OR time block usually require the patient to enter in the hospital, at least on the eve of the surgery’s day, directly to the wards to be prepared to the surgical intervention. The inpatient surgeries scheduled to the other OR time blocks in the first shift usually require the patient to enter in

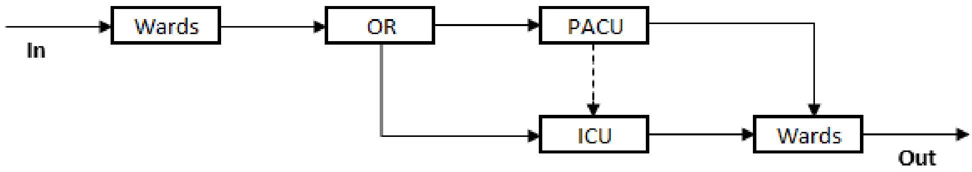

Figure 2.4: Inpatient flow

the hospital at least at 7:30am of the surgery's day, and the inpatient surgeries scheduled to the second shift usually require the patient to enter the hospital at least at 12am of the surgery's day. All the inpatient surgeries that require bowel preparation imply that the patient enters in the hospital at least on the eve of the surgery's day. Only emergency surgical procedures have the patient at the wards at an unexpected time. From the wards, the patient proceeds to the OR, he is moved from the ward type bed to the OR type bed, and prepared with the help of the anesthetist in a small room next to the main room – each room at the OR has a "support room" at its entrance. The patient then crosses to the main room and is exposed to the agreed surgical procedures. When the surgery ends, the patient is usually moved to the post-anesthesia care unit (PACU) to recover from the anesthesia. In special cases, when the health of the patient is considered fragile, he goes to the intensive care unit (ICU). When this is the case, the time spent in this unit is known to be longer than the time spent in the PACU. Even though unusual, it can happen that a patient, after being sent to the PACU, does not recover from the anesthesia as expected and has to be transferred to the ICU. In all the cases, when the patient recovers from the anesthesia, he is sent to the wards to be monitored until he is "fully" recovered from the surgery and thus is able to leave the hospital.

In Figure 2.5 is shown the outpatient flow. Usually the patient enters the hospital a couple of hours before surgery and is sent to the ambulatory care unit (ACU), where the patient is prepared for the surgery: he it asked about the compliance with the preoperative recommendations, the unit's staff verify his exams and the validity of the anesthetic consultation (since it expires every six months). The patient is also asked to change to scrubs – operating room clothes. As it happens in the inpatient flow, the outpatient is moved to the OR, where he is submitted to anesthesia. After leaving the OR, he is moved to the PACU. When something goes as unexpected, the patient is sent to the ICU, but in the great majority of the cases the patient is sent back to the ACU, after recovering from the anesthesia and become awake. The patient that enters in the ICU only leaves to the wards becoming an inpatient. In both cases, when the patient is "fully" recovered from the surgery, he has medical release and is allowed to go with a companion to his residence. As said before, an outpatient goes through the multiple stages in less than a day.

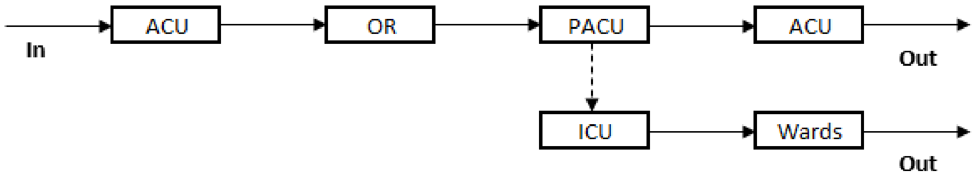

Figure 2.5: Outpatient flow

This hospital adopted weekly surgical planning, that is discussed and finalized on a meeting with the head doctor of the surgical suite and the head nurse of the surgical scheduling department, each Friday at 4pm, for the following week. At this stage, the staff of the surgical scheduling department has confirmed, in the IT platform, the validity of the anesthesia consultation and the preoperative recommendations of all the patients with surgery in the following week. The staff also contact by phone every scheduled patient and confirm with them the realization of the anesthetic consultation in the previous six months, the preoperative recommendations, the realization of the necessary exams, and ask the patients to bring the exams in the day of the surgery. If the patient does not have a valid anesthesia consultation, a consultation is scheduled according to the availability of the patient and the anesthesiologist, and the surgery's date. From the anesthesia consultation, the patient is sent to the several clinical and exam units in order to perform an accurate preoperative evaluation.

The planning can suffer some changes during the course of the week until late afternoon of the surgery's previous day. Emergency surgeries may arise during the course of the day and must take place as soon as there is an available room. This can lead to a delay in the planned surgeries. For this reason, it is important to consider some slacks in the planning. By placing the slacks in an appropriate pattern, it is possible to reduce the impact of the emergency patients in the planned surgeries. Thus, emergency patients are operated as soon as needed, and elective patients are operated when expected, leading to a good service level. The surgical schedule is real-time displayed at the surgeon's sitting room, which is inside the surgical suite. In alternative, OR's staff - surgeons, nurses or administrative assistants - can access online the entire plan in their personal area on the IT platform.

The construction of the weekly surgical plan is a manual procedure that relies on the MSS and requires the contribution of several human resources. This decision level, like the tactical level, requires too much staff time and it is suspected that its outcome is far from being an efficient use of the surgical suite. Nevertheless, the scope of this work is only focused in the tactical level. As a result, this work aims to create a methodology and a model to generate an appropriate MSS to the hospital's surgical workload. Note that it is not guaranteed that the OR time available is sufficient to assign all the surgeons or surgical specialties. Though, based on the data collected from the hospital's historical records, it is possible to show that such requirement is not sufficient to create an unfeasible problem, since the real OR time allocation rate never exceeds 69%.

The MSS problem allocates surgeons or surgical specialties for a specific OR time block, a day and a room, for a weekly planning horizon. In this case study, the OR time block is a portion of a shift. The problem must also take into account structural constraints such as non-overlapping of different surgeons or surgical specialties on the same OR time block, day and room, and non-overlapping of the same surgeon in different rooms on the same OR time block and day (or more generally, at the same shift and day). Figure 2.6 shows examples of both types of overlap. At Monday's morning in room 1 and 2 occurs overlapping of the same surgeon in different rooms on the same shift and day, and at Monday's afternoon in room 3 occurs overlapping of different surgical specialties on the shift, day and room. At Monday's afternoon in room 1 and 2, and at Monday's morning in room 3 there is no overlap of different surgeons on the same OR time block, day and room, since the surgeons are sharing the shift in OR time blocks. This topic will be addressed in the next chapter. The model determines which surgeons have sufficient workflow to be individually allocated to an OR time block. Those surgeons with less workflow must be allocated to an OR time block in a group of surgeons of the same surgical specialty. In this latter case, surgeons do not have an individual OR time block and share OR time with other surgeons from the same surgical specialty. The induction and waking up time is included in the surgery duration, since it is also performed in the OR. The cleaning and disinfecting protocols (fifteen minutes on average) are incorporated in the surgery duration in order to better estimate the weekly time required for each surgeon in terms of OR occupation. Surgery durations are not assumed deterministic, and this is the reason why the model must receive as input a stochastic variable representing the duration of a surgery. The MSS problem also considers daily and weekly operating time limits for each surgeon. The problem has to reflect some hospital's rules, as for example, on each shift, day and room, we can assign up to three surgeons or one surgical specialty. Time is discretized in periods of thirty minutes, generating sixteen time periods in the first shift and fourteen time periods in the second shift. The choice of the time period duration was not difficult, since the increasing of the time periods accuracy would produce a greater complexity in the model and it is not known that it would generate a better practical result at this decision level.

Furthermore, the aim of this work induces four main optimization criteria to the MSS problem:

1. to minimize the workload variability at the care units, in which there are major fluctuations, in order to level the workload as much as possible,

<table>
<tr><td>Room</td><td colspan="4">1</td><td colspan="4">2</td><td colspan="6">3</td></tr>
<tr><td>Day</td><td colspan="4">Monday</td><td colspan="4">Monday</td><td colspan="6">Monday</td></tr>
<tr><td>Shift</td><td>Morning</td><td colspan="3">Afternoon</td><td>Morning</td><td colspan="3">Afternoon</td><td colspan="3">Morning</td><td colspan="3">Afternoon</td></tr>
<tr><td>Surgical specialty's ID</td><td>OPH</td><td colspan="3">ORT</td><td>OPH</td><td>OPH</td><td colspan="2">ORT</td><td>GES</td><td>URO</td><td></td><td>GYN</td><td>GES</td><td></td></tr>
<tr><td>Surgeon's ID</td><td>SRG30</td><td>SRG34</td><td>SRG14</td><td>SRG27</td><td>SRG30</td><td>SRG30</td><td>SRG7</td><td>SRG34</td><td>SRG8</td><td>SRG25</td><td></td><td></td><td></td><td></td></tr>
</table>

Figure 2.6: Example of a MSS, for three rooms, which are open on Monday, with overlapping of surgeons and surgical specialties

2. to minimize the number of assigned rooms per surgical specialty (i.e. to concentrate surgeons that belong to the same surgical specialty as much as possible in the same room),

3. to minimize the deviation of the assignment of each surgical specialty to the difference between the maximum number of available and not individually assigned surgeons that belong to the surgical specialty and the number of the available and not individually assigned surgeons that belong to the specific surgical specialty on the OR time of block under study (i.e. to allocate shifts to surgical specialties when the highest number of available and not individually assigned surgeons that belong to the surgical specialty occur), and

4. to minimize the positive and negative deviations of the weekly duration assigned to surgeons or surgical specialty to the median value of the weekly time used by the surgeon or the surgical specialty in the last trimester (i.e. to renew the MSS based on recent historical data).

These objectives clearly have a conflicting nature. In fact, for instance, when the objective is to minimize the first criterion, it is preferable to assign surgeons with different operating time in a specific shift and day, in order to level the number of patients sent to the recovery units during the week (cycle time). But when the objective is to minimize the second criterion, it is preferable to schedule surgeons from the same specialty in a specific day and room, in order to reduce the number of rooms per surgical specialty. Since the operating time of a surgeon is highly related to his surgical specialty, these objectives in particular are conflictuous.

The mathematical model designed for this MSS problem is detailed in the next chapter, as well as the underlying methodology.

Chapter 3

Model formulation

This chapter introduces the modelling approach and methodology. Some problem requirements that will be formulated as structural constraints, in order to be incorporated in the Mixed-Integer Linear Programming (MILP) model, will be discussed first. Then, the four optimization criteria will be enunciated one at a time, and the restrictions for its mathematical formulation will be established in order to present the criterion's expression. The chapter ends with the setting of the variables' domain. Appendix A presents the notation used and the full model formulation.

3.1 Structural constraints

In this case study, the MSS problem must allocate surgeons or surgical specialties for a specific OR time block, a day and a room, for a weekly planning horizon. Each OR time block is a portion of a specific shift. The model must determine which surgeons have sufficient workflow to be individually allocated to an OR time block, or, in the opposite case, must be allocated to an OR time block in a group of surgeons of the same surgical specialty. The model must incorporate the hospital's rule stating that, on each shift, day and room, up to three surgeons or one surgical specialty can be assigned. Therefore, in Figure 3.1, it can be found an example of a cyclic MSS for a single room, which is open from Monday to Wednesday (in both shifts). In this figure, it is possible to find an example of each type of conceivable allocation. Each shift, day and room can be assigned to one surgeon, and thus the surgeon is individually assigned to an OR time block, a day and a room, with a duration equal to the shift duration (as shown at Monday's morning). Each shift, day and room, can also be assigned to two or three surgeons, and thus each of the surgeons are individually assigned to an OR time block, a day and a room, and the sum of the durations is equal to the shift duration. Thus, the two or three surgeons have to share

<table>
<tr><td>Day</td><td colspan="4">Monday</td><td colspan="4">Tuesday</td><td colspan="4">Wednesday</td></tr>
<tr><td>Shift</td><td>Morning</td><td colspan="3">Afternoon</td><td>Morning</td><td colspan="3">Afternoon</td><td colspan="3">Morning</td><td>Afternoon</td></tr>
<tr><td>Surgical specialty's ID</td><td>OPH</td><td colspan="3">ORT</td><td>GYN</td><td>GES</td><td>URO</td><td></td><td>OPH</td><td colspan="2">ORT</td><td></td></tr>
<tr><td>Surgeon's ID</td><td>SRG30</td><td>SRG34</td><td>SRG14</td><td>SRG27</td><td></td><td>SRG8</td><td>SRG25</td><td></td><td>SRG30</td><td>SRG7</td><td>SRG34</td><td></td></tr>
</table>

Figure 3.1: Example of a cyclic MSS for a room, which is open from Monday to Wednesday

(not necessarily equitably) the shift, day and room. The duration of each of the OR time blocks reflects the portion in which the assigned surgeons should share the shift. The surgeons can be, or not, from the same surgical specialty. An example of surgeons, from the same surgical specialty, individually assigned and sharing a shift in OR time blocks is shown at Monday's afternoon. An example of surgeons, from different surgical specialties, individually assigned and sharing a shift in OR time blocks is shown at Tuesday's afternoon and at Wednesday's morning. Note that the order of the surgeons' ID does not necessarily reflect the order in which they must operate. Surgeons can be individually assigned to more than one OR time block, either sharing or not the shift (as shown at Monday's and Wednesday morning). Alternatively, each shift, day and room can be allocated to a surgical specialty, and thus the surgeons with less workflow are allocated to the OR time block in a group of surgeons of the same surgical specialty. In this case, the duration assigned to the surgical specialty equals the shift capacity (as shown at Tuesday's morning). A shift, day and room can also not be assigned, and then is called a slack. Blank OR time blocks can be used for any surgeon or surgical specialty if needed (namely, each blank OR time block is assigned on a first-come, first-serve basis) as shown at Wednesday's afternoon. In particular, it can be used for emergency surgeries that may arrive.

Concerning the surgeons' individual allocation, the problem must take into account that each surgeon cannot be assigned to two different rooms at same day and OR time block (or, more generally, shift). The mathematical formulation of this requirement is stated in constraint set (3.1). Constraint set (3.1) also prevents a surgeon to be scheduled for a day and shift when he is not available.

$$\sum_{r \in R} x_{srdk} \leq a_{sdk}, \forall s \in S, \, d \in A, \, k \in K, \tag{3.1}$$

where the decision variable

$$x_{srdk} = \begin{cases} 1, \text{ if surgeon } s \text{ obtains an OR time block in room } r \text{ on day } d \text{ and shift } k \\ 0, \text{ otherwise} \end{cases}$$

and parameter

$$a_{sdk} : \begin{cases} 1, \text{ if surgeon } s \text{ is available on day } d \text{ and shift } k \\ 0, \text{ otherwise} \end{cases}$$

The indices s, r, d, and k concern surgeons, rooms, days, and shifts, respectively, whereas S, R, A, and K are the sets of surgeons, rooms, active days during a week, and shifts, respectively.

Constraint set (3.2) prevents the assignment of more than three surgeons in the same room, day and shift.

$$\sum_{s \in S} x_{srdk} \leq 3 \cdot z_{rdk}, \forall r \in R,\ d \in A,\ k \in K, \tag{3.2}$$

where the auxiliary variable

$$z_{rdk} = \begin{cases} 1, \text{ if an OR time block in room } r \text{ on day } d \text{ and shift } k \text{ is assigned to at least one surgeon} \\ 0, \text{ otherwise} \end{cases}$$

Constraint set (3.3) sets the sum of the durations of the OR time blocks assigned to surgeons in each shift, day and room, equal to the shift capacity. When no surgeon is assigned, the sum of the durations of the allocated OR time blocks must be equal to zero.

$$\sum_{s \in S} durb_{srdk} = capacity_{rdk} \cdot z_{rdk}, \forall r \in R,\ d \in A,\ k \in K, \tag{3.3}$$

where the decision variable $durb_{srdk}$ equals the duration (in minutes) of the OR time block allocated to surgeon s in room r on day d and shift k, and parameter $capacity_{rdk}$ provide the total capacity (in minutes) of room r on day d and shift k, which is defined as a multiple of thirty-minute time blocks.

Constraint set (3.4) establishes that, each shift, day and room, can be assigned either to at least one surgeon or one surgical specialty, or not be assigned.

$$z_{rdk} + \sum_{p \in P} y_{prdk} \leq 1, \forall r \in R,\ d \in A,\ k \in K, \tag{3.4}$$

where the decision variable

$$y_{prdk} = \begin{cases} 1, \text{ if surgical specialty } p \text{ is assigned to room } r \text{ on day } d \text{ and shift } k \\ 0, \text{ otherwise} \end{cases}$$

The index p concerns surgical specialties, whereas P is the set of surgical specialties.

Constraint set (3.5) assures that each surgeon only has an allocated OR time block with positive duration when he is assigned for the correspondent shift, room and day.

$$durb_{srdk} \leq capacity_{rdk} \cdot x_{srdk}, \forall s \in S, r \in R, d \in A, k \in K \tag{3.5}$$

Time is discretized in periods of thirty minutes. Constraint set (3.6) states that the duration of each allocated OR time block is a multiple of thirty-minute time blocks.

$$durb_{srdk} = 30 \cdot u_{srdk}, \forall s \in S, r \in R, d \in A, k \in K, \tag{3.6}$$

where the auxiliary variable u_{srdk} equals the (integer) number of thirty-minute time blocks of the OR time block allocated to surgeon s in room r on day d and shift k.

The model considers daily and weekly operating time limits for each surgeon individually assigned, and so the next two sets of constraints define the lower and upper bounds for the daily and weekly working hours (for each surgeon individually assigned), which are also related to surgeon's experience or background. Constraint set (3.7) limits the sum of the duration of the daily allocated OR time blocks, for each surgeon individually assigned, between the minimum and maximum duration (in minutes) that the surgeon can operate each day.

$$min_s^{day} \cdot z_{sd}^{day} \leq \sum_{r \in R} \sum_{k \in K} durb_{srdk} \leq max_s^{day} \cdot z_{sd}^{day}, \forall s \in S, d \in A, \tag{3.7}$$

where the auxiliary variable

$$z_{sd}^{day} = \begin{cases} 1, \text{ if surgeon } s \text{ obtains at least one OR time block on day } d \\ 0, \text{ otherwise} \end{cases}$$

and parameters min_s^{day} and max_s^{day} provide the minimum and maximum duration (as a multiple of a thirty-minute time blocks) that can be daily assigned to surgeon s, respectively.

Likewise, constraint set (3.8) restrains the sum of the duration of the weekly allocated OR time blocks, for each surgeon individually assigned, between the minimum and maximum duration (in minutes) that the surgeon can operate each week.

$$min_s^{week} \cdot z_s^{week} \leq \sum_{r \in R} \sum_{d \in A} \sum_{k \in K} durb_{srdk} \leq max_s^{week} \cdot z_s^{week}, \, \forall s \in S, \tag{3.8}$$

where the auxiliary variable

$$z_s^{week} = \begin{cases} 1, \text{ if surgeon } s \text{ obtains at least one OR time block during the week} \\ 0, \text{ otherwise} \end{cases}$$

and parameters min_s^{week} and max_s^{week} provide the minimum and maximum duration (as a multiple of a thirty-minute time blocks) that can be weekly assigned to surgeon s, respectively.

The decision variable that describe if a surgeon obtains an OR time block at a specific room, day and shift, x_{srdk}, gives the information needed to define the auxiliary variables z_{sd}^{day} and z_s^{week}. Constraint set (3.9) ensures that the variable z_{sd}^{day} takes the right value.

$$\sum_{r \in R} \sum_{k \in K} x_{srdk} \leq |K| \cdot z_{sd}^{day}, \, \forall s \in S, \, d \in A, \tag{3.9}$$

where $|K|$ is the number of elements in set K (namely, the number of shifts considered).

Likewise, constraint set (3.10) ensures that the variable z_s^{week} is well-defined.

$$\sum_{r \in R} \sum_{d \in A} \sum_{k \in K} x_{srdk} \leq |A| \cdot |K| \cdot z_s^{week}, \, \forall s \in S, \tag{3.10}$$

where $|A|$ is the number of elements in set A (namely, the number of active days considered during the week).

In past MSSs[1] of the hospital under study, which were created by the head doctor of the surgical theatre, the assignment rate was at least 80%. The assignment of surgical specialties never exceeds 3% of the total OR time. In the literature, it was found evidence that most industry sources believe that an OR utilization between 75% and 80% is acceptable. Johnson and Johnson desires to have an OR utilization of 75% for individually assigned surgeons, and an OR utilization of 80% in total, including surgical specialties allocation (Hosseini & Taaffe 2015). An OR utilization over 80% requires extremely good supporting systems (particularly with respect to bed occupancy, pre-admissions testing and the PACU access) (Company 2001). Thus, it was decided to consider at least an OR utilization of 75% for surgeons individually

[1]MSSs used, in the operating theatre under study, in a trimester of 2013, in the last trimester of 2014, and in the first trimester of 2015.

assigned.

$$\sum_{s\in S}\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} durb_{srdk} \geq 0.75 \cdot \sum_{r\in R}\sum_{d\in A}\sum_{k\in K} capacity_{rdk} \tag{3.11}$$

Constraint (3.12) demands an OR utilization of at least 80% for surgeons individually assigned and surgical specialties' allocation.

$$\sum_{r\in R}\sum_{d\in A}\sum_{k\in K}(\sum_{s\in S} durb_{srdk} + capacity_{rdk} \cdot \sum_{p\in P} y_{prdk}) \geq 0.8 \cdot \sum_{r\in R}\sum_{d\in A}\sum_{k\in K} capacity_{rdk} \tag{3.12}$$

3.2 Optimization criteria

First optimization criterion

The first optimization criterion aims to minimize the workload variability at the care units in which there are major fluctuations, in order to level the workload as much as possible. This is done by minimizing the weighted peaks in the expected number of surgical patients sent to those hospitalization units, and in the associated absolute deviation.

To state the mathematical formulation of the first criterion, the number of patients that an individual surgeon or a group of surgeons from the same surgical specialty can operate on each day must be defined. Since the surgeries' duration is not assumed deterministic, the number of operated patients on each day varies with the duration of the surgeries performed by the assigned surgeon or by the group of surgeons from the allocated surgical specialty.

The stochastic variables (considered as inputs) DUR_s^{SRG} and DUR_p^{SPC} provide, respectively, the duration of a surgery (in minutes) performed by surgeon s or by a surgeon from the surgical specialty p, including the induction and waking time, and the cleaning procedures. The empirical distribution of the stochastic variables was fitted from the collected data. The variable values were aggregated into classes using Sturges' rule, and then the corresponding values were incorporated in the model as inputs. By aggregating the variable values using Sturges' rule, the types of procedure that the surgeon performs are being grouped by duration, and at best, all the occurrences of a procedure are being isolated in each class. Figure 3.2 shows an example of an empirical distribution of the stochastic variable DUR_s^{SRG}, after their values were aggregated into classes by Sturges' rule, and how parameters dur_{sq}^{SRG} and $pdur_{sq}^{SRG}$ were extracted in order to be incorporated in the model. Parameter dur_{sq}^{SRG} represents the midpoint of class q, with $q \in Q_s^{SRG}$, of the stochastic variable DUR_s^{SRG}, while parameter $pdur_{sq}^{SRG}$ represents the corresponding probability ($pdur_{sq}^{SRG} = P(DUR_s^{SRG} = dur_{sq}^{SRG})$). So, q is the index for the classes, and Q_s^{SRG} are the sets of classes of the stochastic variables

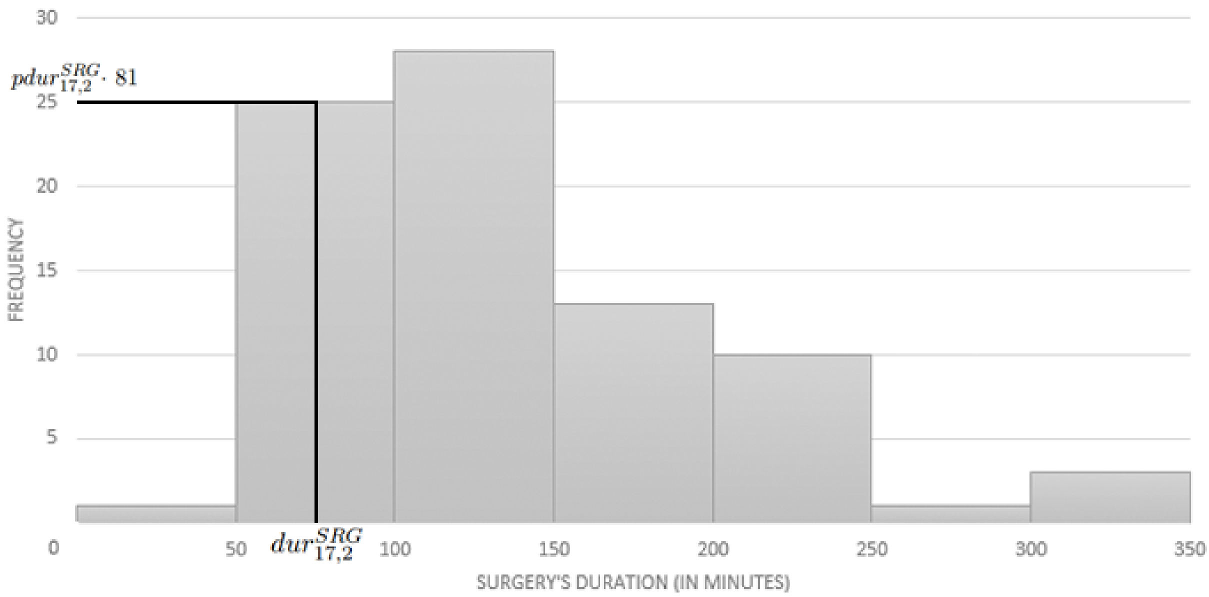

Figure 3.2: Histogram of the duration of a surgery performed by surgeon 17 (81 surgeries performed), after aggregating the values using Sturges' rule, and its relation with the parameters dur_{sq}^{SRG} and $pdur_{sq}^{SRG}$.

DUR_s^{SRG}. The total number of surgeries performed by the surgeon analyzed in Figure 3.2 is 81, and the number of surgeries with duration between 50 and 100 minutes is 25. The relation is stated as $pdur_{17,2}^{SRG} = P(DUR_{17}^{SRG} = 75) \approx 25\ /\ 81$, since $pdur_{17,2}^{SRG}$ is approximated by the relative frequency of a surgery's duration, performed by surgeon 17, belonging to class 2 of the stochastic variable DUR_{17}^{SRG}. It was used the same reasoning for parameters dur_{pq}^{SPC} and $pdur_{pq}^{SPC}$. Parameter dur_{pq}^{SPC} represents the midpoint of class q, with $q \in Q_p^{SPC}$, of the stochastic variable DUR_p^{SPC}, while parameter $pdur_{pq}^{SPC}$ represents the corresponding probability ($pdur_{pq}^{SPC} = P(DUR_p^{SPC} = dur_{pq}^{SPC})$). So, Q_p^{SPC} are the sets of classes of the stochastic variables DUR_p^{SPC}.

At first, a fitting attempt was made in order to find a named probability distribution (e.g. exponential distribution) that suits the collected historical data (concerning the duration of the surgeries), but unfortunately it was impossible to find one. In the best fitting attempt not even 50% of the surgeons had their surgery's duration accurately estimated, and so it was decided to use the empirical distribution with the variable values aggregated into classes. We suspect that the attempt to find a good fitting based on a named probability distribution failed, because the duration of the surgeries varies significantly with the surgeon's surgical specialty, type of surgeries that the surgeon performs, and surgical procedures or techniques in which he feels comfortable. Unknown factors such us as surgeon's inexperience may also contribute to the behavior of the empirical function. Even if the major factors were known, given the sample size, after disaggregating the surgeries performed by type, procedure, execution technique, or other, the data is insufficient to provide reliable conclusions.

The auxiliary variables n_{sdq}^{SRG} represent the number of patients that surgeon s can operate on day d, if the value of the stochastic variable DUR_s^{SRG} belongs to class q, while the auxiliary variables n_{pdq}^{SPC} represent the number of patients that a surgeon of surgical specialty p can operate on day d, if the value of the stochastic variable DUR_p^{SPC} belongs to class q. Note that the auxiliary variables n_{sdq}^{SRG} and n_{pdq}^{SPC} depend also on the duration of the OR time block that is allocated to the surgeon or to the surgical specialty on a given day.

Constraint set (3.13) calculates the number of surgeries that can be performed, by the individually assigned surgeon, during the allocated OR time blocks considering different scenarios for surgery's duration, and so reflects the calculation formula for the surgeon's analysis based on the described methodology.

$$n_{sdq}^{SRG} = \sum_{r \in R} \sum_{k \in K} \frac{durb_{srdk}}{dur_{sq}^{SRG}}, \forall s \in S,\, d \in A,\, q \in Q_s^{SRG} \tag{3.13}$$

Constraint set (3.14) calculates the number of surgeries that can be performed, by a surgeon from the allocated surgical specialty, during the assigned OR time blocks considering different scenarios for surgery's duration. Since the duration of an OR block allocated to a surgical specialty is always equal to the capacity of the shift, n_{pdq}^{SPC} only varies with the latter and with the belonging class of the duration of a surgery performed within the same surgical specialty.

$$n_{pdq}^{SPC} = \sum_{r \in R} \sum_{k \in K} \frac{capacity_{rdk} \cdot y_{prdk}}{dur_{pq}^{SPC}}, \forall p \in P,\, d \in A,\, q \in Q_p^{SPC} \tag{3.14}$$

The constraint set (3.14) that reflects the calculation formula for the surgical specialty's analysis, like in the previous constraint set, sums up the OR time blocks, for all rooms and shifts, that are allocated to each surgical specialty in a given day. It is possible that a surgeon or a surgical specialty has an OR time block in the morning shift and another one in the afternoon shift. By summing up the durations of each allocated OR time block in those constraint sets, the formulation is not being affected negatively. Surgical specialties can also have OR time blocks in more than one room. By summing up the durations of all allocated shifts, the constraint sets (3.15) to (3.22) are actually being simplified.

The contribution of allocating an OR time block to surgeon s or to surgical specialty p on day d to the mean number of patients sent to the hospitalization unit h is represented by the auxiliary variables m_{sdh}^{SRG} and m_{pdh}^{SPC}, respectively. Both types of auxiliary variables are defined considering the conditional mean calculation formula and some of its properties. So, consider that U_{sdh} denotes the stochastic variable representing the number of patients sent to the hospitalization unit h, after being operated by surgeon s on day d, and N_{sd} denotes the stochastic variable representing the number of patients operated by surgeon s on day d. The mean number of patients sent to each hospitalization unit by each surgeon on each day is given through $E(U_{sdh})$. Therefore, using the law of total expectation

$$
\begin{aligned}
m_{sdh}^{SRG} &= E(U_{sdh}) \\
&= E(E[U_{sdh}|N_{sd}]) \\
&= \sum_{q \in Q_s^{SRG}} P(N_{sd} = n_{sdq}^{SRG}) \cdot E(U_{sdh}|N_{sd} = n_{sdq}^{SRG}) \\
&= \sum_{q \in Q_s^{SRG}} pdur_{sq}^{SRG} \cdot \left(phu_{sh}^{SRG} \cdot n_{sdq}^{SRG}\right), \forall s \in S,\, d \in A,\, h \in H,
\end{aligned}
$$

where parameter phu_{sh}^{SRG} equals the probability for a patient to be sent to the hospitalization unit h, after being operated by surgeon s, and the index h and set H refer to hospitalization units.

The following constraint set, which calculates the contribution, to the mean number of patients sent to each hospitalization unit, of allocating an OR time block to a surgeon on a specific day, will thus be added to the model.

$$m_{sdh}^{SRG} = \sum_{q \in Q_s^{SRG}} pdur_{sq}^{SRG} \cdot phu_{sh}^{SRG} \cdot n_{sdq}^{SRG}, \forall s \in S,\, d \in A, h \in H \tag{3.15}$$

Using the same reasoning, it is possible to formulate the equivalent constraint set for the contribution, to the mean number of patients sent to each hospitalization unit, of allocating an OR time block to a surgical specialty on a specific day. The constraint set is as follows:

$$m_{pdh}^{SPC} = \sum_{q \in Q_p^{SPC}} pdur_{pq}^{SPC} \cdot phu_{ph}^{SPC} \cdot n_{pdq}^{SPC}, \forall p \in P,\, d \in A,\, h \in H, \tag{3.16}$$

where parameter phu_{ph}^{SPC} equals the probability for a patient to be sent to the hospitalization unit h, after being operated by a surgeon of surgical specialty p.

The variance and the standard deviation are the most used measures of dispersion. However, the absolute deviation (from the mean) will be considered, because it allows a linear formulation of the problem.

The contribution of allocating an OR time block to surgeon s on day d to the absolute deviation from the mean of the number of patients sent to the hospitalization unit h, if the value of the stochastic variable DUR_s^{SRG} belongs to class q, is represented by the auxiliary variable d_{sdhq}^{SRG}. The calculation formula of the absolute deviation is presented below, and it will be used as a measure of dispersion for the surgeon's analysis at first.

$$\begin{aligned} d_{sdhq}^{SRG} &= \left|E(U_{sdh}|N_{sd} = n_{sdq}^{SRG}) - m_{sdh}^{SRG}\right| \\ &= \left|phu_{sh}^{SRG} \cdot n_{sdq}^{SRG} - m_{sdh}^{SRG}\right|, \forall s \in S,\, d \in A,\, h \in H,\, q \in Q_s^{SRG} \end{aligned}$$

These expressions are linearized through constraint sets (3.17) and (3.18). These constraint sets calculate the absolute deviation from the mean of the number of patients sent to each hospitalization unit concerning the individually assignment of surgeons.

$$d_{sdhq}^{SRG} \geq phu_{sh}^{SRG} \cdot n_{sdq}^{SRG} - m_{sdh}^{SRG}, \forall s \in S,\, d \in A,\, h \in H,\, q \in Q_s^{SRG} \tag{3.17}$$

$$d_{sdhq}^{SRG} \geq m_{sdh}^{SRG} - phu_{sh}^{SRG} \cdot n_{sdq}^{SRG}, \forall s \in S,\, d \in A,\, h \in H,\, q \in Q_s^{SRG} \tag{3.18}$$

Using the same reasoning, it is possible to formulate the equivalent constraint set concerning the assignment of surgical specialties, being the auxiliary variable d_{pdhq}^{SPC} the contribution of allocating an OR time block to surgical specialty p on day d to the absolute deviation of the number of patients sent to the hospitalization unit h, if the value of the stochastic variable DUR_p^{SPC} belongs to class q. The constraint sets to add to the model are as follows:

$$d_{pdhq}^{SPC} \geq phu_{ph}^{SPC} \cdot n_{pdq}^{SPC} - m_{pdh}^{SPC}, \forall p \in P,\, d \in A,\, h \in H,\, q \in Q_p^{SPC} \tag{3.19}$$

$$d_{pdhq}^{SPC} \geq m_{pdh}^{SPC} - phu_{ph}^{SPC} \cdot n_{pdq}^{SPC}, \forall p \in P,\, d \in A,\, h \in H,\, q \in Q_p^{SPC} \tag{3.20}$$

Note that constraint sets (3.17) to (3.20), unlike constraint sets (3.15) and (3.16), also vary with the belonging class of the duration of a surgery performed by each surgeon individually assigned or by each surgical specialty.

The mean number of patients sent to the hospitalization unit h on day d is given by the auxiliary variable $mean_{dh}$, while the weighted absolute deviation of the number of patients sent to the hospitalization unit h on day d is given by the auxiliary variable ad_{dh}.

It is assumed that the two types of stochastic variables (DUR_s^{SRG} and DUR_p^{SPC}) are independent, since only the surgeons that are not individually assigned should share the OR time blocks allocated to the correspondent surgical specialty.

The mean number of patients sent, on each day, to each hospitalization unit is given by the sum of the mean number of patients sent to the hospitalization unit, on each OR time block allocated to a surgeon or surgical specialty (contribution). The mathematical formulation of the constraint set is as follows:

$$mean_{dh} = \sum_{s \in S} m_{sdh}^{SRG} + \sum_{p \in P} m_{pdh}^{SPC}, \, \forall d \in A, \, h \in H \tag{3.21}$$

The constraint set which defines the absolute deviation of the number of patients sent, on each day, to each hospitalization unit is obtained using the same reasoning. However, it was decided to weight the absolute deviations' contributions by the corresponding probability of a surgery's duration. Note that, as referred to before, in this constraint set the contribution variables are indexed in the classes of the corresponding stochastic variables.

$$ad_{dh} = \sum_{s \in S} \sum_{q \in Q_s^{SRG}} pdur_{sq}^{SRG} \cdot d_{sdhq}^{SRG} + \sum_{p \in P} \sum_{q \in Q_p^{SPC}} pdur_{pq}^{SPC} \cdot d_{pdhq}^{SPC}, \, \forall d \in A, \, h \in H \tag{3.22}$$

The maximum or peak mean number of patients sent to the hospitalization unit h over all active days, $\overline{mean_h}$, and the maximum or peak weighted absolute deviation of the number of patients sent to the hospitalization unit h over all active days, $\overline{ad_h}$ can now be defined. These auxiliary variables correspond to the maximum of the auxiliary variables $mean_{dh}$ $(d \in A)$ since the peak mean number of patients sent to a hospitalization unit over all active days has to happen in one of the active days. The corresponding linearized constraint set is as follows:

$$mean_{dh} \leq \overline{mean_h}, \, \forall d \in A, \, h \in H \tag{3.23}$$

Using the same type of reasoning, the linearized constraint set is obtained to define the peak weighted absolute deviation of the number of patients sent to each hospitalization unit.

$$ad_{dh} \leq \overline{ad_h}, \, \forall d \in A, \, h \in H \tag{3.24}$$

Thus, constraint sets (3.23) and (3.24) provide the link with the first optimization criterion of the objective function by imposing the expected number of patients sent to each hospitalization unit and the respective weighted absolute deviation not to exceed the associated peak. Since the study goal is to minimize then the mathematical formulation of the first criterion is as follows:

$$\text{Minimize } \sum_{h \in H} Wmean_h \cdot \overline{mean_h} + \sum_{h \in H} Wad_h \cdot \overline{ad_h}, \tag{3.25}$$

where parameters $Wmean_h$ equal the relative importance of leveling the mean number of patients sent to the hospitalization unit h, while parameters Wad_h equal the relative importance of leveling the corresponding weighted absolute deviation.

Example 3.2.1. Considering the data presented in Figure 3.2, and so the different scenarios for surgery's duration, the number of surgeries that can be performed, by the individually assigned surgeon, during the allocated OR time blocks (constraint set (3.13)) will be calculated. The contribution to the mean number of patients sent to each hospitalization unit (constraint set (3.15)), and the correspondent absolute deviation, if the value of the stochastic variable DUR_{17}^{SRG} belongs to class q (constraint sets (3.17) and (3.18)) will also be calculated. Imagine that: there are seven ORs in the surgical suite ($R = \{1, \ldots, 7\}$); the ORs are open from Monday to Friday, so five days during the week ($A = \{1, \ldots, 5\}$); all the ORs have two shifts ($K = \{1, 2\}$) per day; and the hospital has two hospitalization units, where patients recover after the surgery ($H = \{1, 2\}$). The duration of the surgeries performed by surgeon 17 were aggregated in seven classes, so $Q_{17}^{SRG} = \{1, \ldots, 7\}$. Table 3.1 summarizes the data on histogram of Figure 3.2. It presents the values of parameters $dur_{17\,q}^{SRG}$ and $pdur_{17\,q}^{SRG}$, for each of the seven classes.

Also imagine that surgeon 17 has two weekly OR time blocks in room 1 on day 3 (Wednesday), one in the morning shift, and another in the afternoon shift. So, $x_{17,1,3,1} = 1$, and $x_{17,1,3,2} = 1$, while the remaining variables $x_{17,r,d,k}$ equal zero. On shift 1, surgeon 17 has 180 minutes, and on shift 2, he has 210 minutes. Then, $durb_{17,1,3,1}^{SRG} = 180$, and $durb_{17,1,3,2}^{SRG} = 210$, while the remaining variables $durb_{17,r,d,k}^{SRG}$ equal zero. In about 4% of the cases, the patient has to recover in hospitalization unit 1, after being operated by surgeon 17, while in about 96% of the cases, the patient recovers in hospitalization unit 2. Therefore, the number of surgeries that can be performed, by the individually assigned surgeon 17, during the allocated OR time blocks is given by

$$n_{17,d,q}^{SRG} = \begin{cases} \sum\limits_{r \in R} \sum\limits_{k \in K} \frac{durb_{17,r,d,k}}{dur_{17,q}^{SRG}},\ \forall d = 3,\ q \in Q_{17}^{SRG} \quad \text{(I)} \\ 0,\ \forall d \in A - \{3\},\ q \in Q_{17}^{SRG} \quad \text{(II)} \end{cases}$$

Rewriting constraint set (I), and substituting the previously presented parameters and decision variables,

Table 3.1: Parameters' values of the midpoint of each class q of the stochastic variable DUR_{17}^{SRG}, $dur_{17,q}^{SRG}$, and the corresponding probability, $pdur_{17,q}^{SRG}$

q	1	2	3	4	5	6	7
$dur_{17,q}^{SRG}$	25	75	125	175	225	275	325
$pdur_{17,q}^{SRG}$	0.012	0.309	0.346	0.161	0.123	0.012	0.037

$$\begin{cases} n^{SRG}_{17,3,1} = \frac{durb_{17,1,3,1}+durb_{17,1,3,2}}{dur^{SRG}_{17,1}} = \frac{180+210}{25} = 15.6 \\ n^{SRG}_{17,3,2} = \frac{durb_{17,1,3,1}+durb_{17,1,3,2}}{dur^{SRG}_{17,2}} = \frac{180+210}{75} = 5.2 \\ n^{SRG}_{17,3,3} = \frac{durb_{17,1,3,1}+durb_{17,1,3,2}}{dur^{SRG}_{17,3}} = \frac{180+210}{125} = 3.12 \\ n^{SRG}_{17,3,4} = \frac{durb_{17,1,3,1}+durb_{17,1,3,2}}{dur^{SRG}_{17,4}} \approx \frac{180+210}{175} = 2.229 \\ n^{SRG}_{17,3,5} = \frac{durb_{17,1,3,1}+durb_{17,1,3,2}}{dur^{SRG}_{17,5}} \approx \frac{180+210}{225} = 1.733 \\ n^{SRG}_{17,3,6} = \frac{durb_{17,1,3,1}+durb_{17,1,3,2}}{dur^{SRG}_{17,6}} \approx \frac{180+210}{275} = 1.418 \\ n^{SRG}_{17,3,7} = \frac{durb_{17,1,3,1}+durb_{17,1,3,2}}{dur^{SRG}_{17,7}} = \frac{180+210}{325} = 1.2 \end{cases}$$

The contribution to the mean number of patients sent to each hospitalization unit of allocating an OR time block to surgeon 17 is given by

$$m^{SRG}_{17,d,h} = \begin{cases} \sum\limits_{q \in Q^{SRG}_{17}} pdur^{SRG}_{17,q} \cdot phu^{SRG}_{17,h} \cdot n^{SRG}_{17,d,q}, \ \forall d = 3, h \in H \ \ \text{(III)} \\ 0, \ \forall d \in A - \{3\}, \ h \in H \ \ \text{(IV)} \end{cases}$$

Rewriting constraint set (III),

$$\begin{cases} m^{SRG}_{17,3,1} = pdur^{SRG}_{17,1} \cdot phu^{SRG}_{17,1} \cdot n^{SRG}_{17,3,1} + pdur^{SRG}_{17,2} \cdot phu^{SRG}_{17,1} \cdot n^{SRG}_{17,3,2} + pdur^{SRG}_{17,3} \cdot phu^{SRG}_{17,1} \cdot n^{SRG}_{17,3,3} + \\ \qquad + pdur^{SRG}_{17,4} \cdot phu^{SRG}_{17,1} \cdot n^{SRG}_{17,3,4} + pdur^{SRG}_{17,5} \cdot phu^{SRG}_{17,1} \cdot n^{SRG}_{17,3,5} + pdur^{SRG}_{17,6} \cdot phu^{SRG}_{17,1} \cdot n^{SRG}_{17,3,6} + \\ \qquad + pdur^{SRG}_{17,7} \cdot phu^{SRG}_{17,1} \cdot n^{SRG}_{17,3,7} \\ m^{SRG}_{17,3,2} = pdur^{SRG}_{17,1} \cdot phu^{SRG}_{17,2} \cdot n^{SRG}_{17,3,1} + pdur^{SRG}_{17,2} \cdot phu^{SRG}_{17,2} \cdot n^{SRG}_{17,3,2} + pdur^{SRG}_{17,3} \cdot phu^{SRG}_{17,2} \cdot n^{SRG}_{17,3,3} + \\ \qquad + pdur^{SRG}_{17,4} \cdot phu^{SRG}_{17,2} \cdot n^{SRG}_{17,3,4} + pdur^{SRG}_{17,5} \cdot phu^{SRG}_{17,2} \cdot n^{SRG}_{17,3,5} + pdur^{SRG}_{17,6} \cdot phu^{SRG}_{17,2} \cdot n^{SRG}_{17,3,6} + \\ \qquad + pdur^{SRG}_{17,7} \cdot phu^{SRG}_{17,2} \cdot n^{SRG}_{17,3,7} \end{cases}$$

And substituting the previously presented parameters and decision variables,

$$\begin{cases} m^{SRG}_{17,3,1} \approx 0.012 \cdot 0.040 \cdot 15.6 + 0.309 \cdot 0.040 \cdot 5.2 + 0.346 \cdot 0.040 \cdot 3.12 + 0.161 \cdot 0.040 \cdot 2.229 + \\ \qquad + 0.123 \cdot 0.040 \cdot 1.733 + 0.012 \cdot 0.040 \cdot 1.418 + 0.037 \cdot 0.040 \cdot 1.2 \\ \qquad \approx \ 0.140 \\ m^{SRG}_{17,3,2} \approx 0.012 \cdot 0.960 \cdot 15.6 + 0.309 \cdot 0.960 \cdot 5.2 + 0.346 \cdot 0.960 \cdot 3.12 + 0.161 \cdot 0.960 \cdot 2.229 + \\ \qquad + 0.123 \cdot 0.960 \cdot 1.733 + 0.012 \cdot 0.960 \cdot 1.418 + 0.037 \cdot 0.960 \cdot 1.2 \\ \qquad \approx \ 3.367 \end{cases}$$

The contribution to the absolute deviation of the number of patients sent to each hospitalization unit, if the value of the stochastic variable DUR_{17}^{SRG} belongs to class q, of allocating an OR time block to surgeon 17 is given by

$$\begin{cases} d_{17,d,h,q}^{SRG} = |phu_{sh}^{SRG} \cdot n_{sdq}^{SRG} - m_{sdh}^{SRG}|, \forall d = 3,\ h \in H,\ q \in Q_{17}^{SRG} \ \ \text{(V)} \\ d_{17,d,h,q}^{SRG} = 0,\ \forall d \in A - \{3\},\ h \in H,\ q \in Q_{17}^{SRG} \ \ \text{(VI)} \end{cases}$$

Rewriting constraint set (V), and substituting the previously presented parameters and decision variables,

$$\begin{cases} d_{17,3,1,1}^{SRG} \approx |0.040 \cdot 15.6 - 0.140| = 0.484 \\ d_{17,3,1,2}^{SRG} \approx |0.040 \cdot 5.2 - 0.140| = 0.068 \\ d_{17,3,1,3}^{SRG} \approx |0.040 \cdot 3.12 - 0.140| \approx 0.015 \\ d_{17,3,1,4}^{SRG} \approx |0.040 \cdot 2.229 - 0.140| \approx 0.051 \\ d_{17,3,1,5}^{SRG} \approx |0.040 \cdot 1.733 - 0.140| \approx 0.071 \\ d_{17,3,1,6}^{SRG} \approx |0.040 \cdot 1.418 - 0.140| \approx 0.083 \\ d_{17,3,1,7}^{SRG} \approx |0.040 \cdot 1.2 - 0.140| = 0.092 \\ d_{17,3,2,1}^{SRG} \approx |0.960 \cdot 15.6 - 3.367| = 11.609 \\ d_{17,3,2,2}^{SRG} \approx |0.960 \cdot 5.2 - 3.367| = 1.625 \\ d_{17,3,2,3}^{SRG} \approx |0.960 \cdot 3.12 - 3.367| \approx 0.372 \\ d_{17,3,2,4}^{SRG} \approx |0.960 \cdot 2.229 - 3.367| \approx 1.227 \\ d_{17,3,2,5}^{SRG} \approx |0.960 \cdot 1.733 - 3.367| \approx 1.703 \\ d_{17,3,2,6}^{SRG} \approx |0.960 \cdot 1.418 - 3.367| \approx 2.006 \\ d_{17,3,2,7}^{SRG} \approx |0.960 \cdot 1.2 - 3.367| = 2.215 \end{cases}$$

After replicate the calculations presented for the remaining surgeons, and surgical specialties, the results can be summed up, and then constraints (3.21) to (3.24) inclusive can be achieved.

□

Second optimization criterion

The second optimization criterion aims to concentrate surgeons that belong to the same surgical specialty as much as possible in the same room, and thus the minimization of the number of assigned rooms per surgical specialty is considered.

The binary auxiliary variable that express if at least a surgeon of a specific surgical specialty obtains an OR time block in a specific room is defined bellow.

$$room_{pr} = \begin{cases} 1, \text{ if at least one surgeon of surgical specialty } p \text{ obtains an OR time block in room } r \\ 0, \text{ otherwise} \end{cases}$$

The mathematical expression (3.26) states that if a surgeon of a specific surgical specialty p or the specific surgical specialty p is not assigned to a specific room, then the binary auxiliary variable $room_{pr}$ for that surgical specialty p and room has value zero, but when it is assigned to at least one surgeon that belongs to the specific surgical specialty p, then the binary auxiliary variable $room_{pr}$ has to take value one. It is necessary to set an upper limit in order to be able to properly define the inequality. The upper limit set was found considering that all the surgeons of the specific surgical specialty p and the surgical specialty itself could have an OR time block at the same specific room r on each active day and shift. The upper limit can be found on the right-hand side of the inequality as coefficient. The inequality follows.

$$\sum_{d \in A} \sum_{k \in K} \left(\sum_{s \in S} x_{srdk} \cdot b_{sp} + y_{prdk} \right) \leq |A| \cdot |K| \cdot \left(\sum_{s \in S} b_{sp} + 1 \right) \cdot room_{pr}, \; \forall p \in P, \; r \in R, \tag{3.26}$$

where parameter

$$b_{sp} : \begin{cases} 1, \text{ if surgeon } s \text{ belongs to surgical specialty } p \\ 0, \text{ otherwise} \end{cases}$$

It is assumed that each surgeon belongs to only one surgical specialty.

Constraint set (3.26) determines which rooms are used by each of the surgical specialties (or surgeons of the surgical specialty), and so it ensures that the second criterion of the objective function obtains the proper value. The mathematical formulation of the second criterion of the objective function is as follows:

$$\text{Minimize} \; \sum_{p \in P} Wroom_p \cdot \sum_{r \in R} room_{pr}, \tag{3.27}$$

where parameter $Wroom_p$ equals the relative importance of concentrating surgeons that belong to surgical specialty p in the same room.

Third optimization criterion

The third optimization criterion intends to force the assignment of the surgical specialty's OR time blocks to be on the days and shifts with the highest number of available and not individually assigned surgeons that belong to the surgical specialty.

Firstly, the maximum number of available and not individually assigned surgeons that belong to the specific surgical specialty p, for each surgical specialty, is defined. Since this maximum has to occur on at least one day and shift, the linearized constraint set that defines it follows. Note that the expression on the right-hand side of the inequality represent the number of available and not individually assigned surgeons that belong to the surgical specialty p, since z_s^{week} mathematically express if a surgeon obtains at least one OR time block weekly.

$$Max_p \geq \sum_{s \in S} b_{sp} \cdot a_{sdk} \cdot (1 - z_s^{week}), \forall p \in P, d \in A, k \in K \tag{3.28}$$

Then, the linearized constraint is formulated as an equality constraint by decrementing an integer deviation variable, $Idev_{pdk}$, which equals the difference between the maximum number of available and not individually assigned surgeons that belong to the surgical specialty p and the number of available and not individually assigned surgeons that belong to the surgical specialty p on day d and shift k.

$$Max_p - Idev_{pdk} = \sum_{s \in S} b_{sp} \cdot a_{sdk} \cdot (1 - z_s^{week}), \forall p \in P, d \in A, k \in K \tag{3.29}$$

Next, the auxiliary variable $Idev_{pdk}$ is considered in order to define the binary deviation variable dev_{pdk}^{-}, which has to take value 1, if the maximum number of available and not individually assigned surgeons that belong to surgical specialty p does not occur on day d and shift k, and may take value 0, otherwise.

$$Idev_{pdk} \leq \sum_{s \in S} b_{sp} \cdot dev_{pdk}^{-}, \forall p \in P, d \in A, k \in K \tag{3.30}$$

Constraint set (3.31),

$$\sum_{r \in R} y_{prdk} + dev_{pdk}^{-} - dev_{pdk}^{+} = 1, \forall p \in P, d \in A, k \in K \tag{3.31}$$

with the help of the third optimization criterion of the objective function, force the auxiliary variable dev_{pdk}^{+} to assume value 0, which means that the surgical specialty p is assigned on day d and shift k only if the maximum number of available and not individually assigned surgeons of the surgical specialty occurs on the specific OR time block, and value 1, otherwise. In other words, it causes the decision variable y_{prdk} to assume value 1, if the corresponding maximum occurs on the day and shift under analysis (dev_{pdk}^{-} equals 0), and value 0, otherwise (dev_{pdk}^{-} equals 1). Note that dev_{pdk}^{-} will also assume value 1, if the maximum number of available and not individually assigned surgeons that belong to surgical specialty p occurs, but still there is no interest in assigning the surgical specialty to the specific OR time block.

The mathematical formulation of the third optimization criterion is as follows:

$$\text{Minimize} \sum_{p\in P} Wdev_p \cdot \sum_{d\in A}\sum_{k\in K} dev^{+}_{pdk}, \tag{3.32}$$

where parameter $Wdev_p$ equals the relative importance of allocating the surgical specialty p on the day and shift in which the highest number of not individually assigned surgeons, that belong to the same surgical specialty, is available.

Additionally, each surgical specialty can be weekly assigned to a maximum of an OR time block, or cannot be assigned at all, if the corresponding maximum, Max_p, takes value 0. The linearized constraint set follows.

$$\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} y_{prdk} \leq 1, \forall p \in P \tag{3.33}$$

$$\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} y_{prdk} \leq Max_p, \forall p \in P \tag{3.34}$$

Fourth optimization criterion

The fourth and last optimization criterion intends to obtain the weekly duration of the OR time blocks assigned to surgeons or surgical specialties to be as close as possible to the median value of the weekly time used by the corresponding surgeons or surgical specialties in the last trimester. The goal is to minimize the positive and negative deviations of the weekly duration assigned to every surgeon or surgical specialty to the median value of the weekly time used by the surgeon or the surgical specialty in the last three months. Thus the MSS is being renew based on the recent historical data.

Let us start with the surgeon's analysis. The sum of the duration of the OR time blocks allocated to each surgeon at any room, day and shift must be as close as possible to the median value of the associated weekly time used by the surgeon in the last trimester. The mathematical formulation follows.

$$\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} durb_{srdk} + dev^{SRG}_{s-} - dev^{SRG}_{s+} = median_s, \forall s \in S, \tag{3.35}$$

where the auxiliary variables dev^{SRG}_{s-} and dev^{SRG}_{s+} represent the negative and positive deviation of the weekly duration assigned to surgeon s to the median value of the weekly time used by the surgeon in the last trimester, respectively, and parameter $median_s$ equals the median value of the weekly time (in minutes) used by surgeon s in the last trimester.

Using the same reasoning, it is possible to formulate the equivalent constraint set for surgical specialty's analysis. The constraint set to add to the model is shown below.

$$\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} capacity_{rdk}\cdot y_{prdk} + dev^{SPC}_{p-} - dev^{SPC}_{p+} = \sum_{s\in S} b_{sp}\cdot median_s\cdot(1-z^{week}_s),\ \forall p\in P, \tag{3.36}$$

where the auxiliary variables dev^{SPC}_{p-} and dev^{SPC}_{p+} represent the negative and positive deviation of the weekly duration assigned to surgical specialty p to the median value of the weekly time used, in the last trimester, by the surgeons from the same surgical specialty with no individual assignment to OR time blocks, respectively. Remember that the OR time blocks allocated to surgical specialties have the duration of the corresponding shift.

Constraint sets (3.32) and (3.33), in line with the forth optimization criterion of the objective function, demand the duration allocated to each surgeon individually assigned or surgical specialty (surgeons not individually assigned), respectively, to be as close as possible to the median value of the associated weekly time used by the surgeon or surgeons in the last trimester. Thus, the mathematical formulation of the forth optimization criterion follows.

$$\begin{aligned}\text{Minimize } &\sum_{s\in S} Wdev^{SRG}_{s-}\cdot dev^{SRG}_{s-} + \sum_{s\in S} Wdev^{SRG}_{s+}\cdot dev^{SRG}_{s+}\\ &+\sum_{p\in P} Wdev^{SPC}_{p-}\cdot dev^{SPC}_{p-} + \sum_{p\in P} Wdev^{SPC}_{p+}\cdot dev^{SPC}_{p+},\end{aligned} \tag{3.37}$$

where parameters $Wdev^{SRG}_{s-}$ and $Wdev^{SRG}_{s+}$ equal the relative importance of diverging negatively and positively the weekly duration assigned to surgeon s to the median value of the weekly time used in the last trimester, respectively, and parameters $Wdev^{SPC}_{p-}$ and $Wdev^{SPC}_{p+}$ equal the relative importance of diverging negatively and positively the weekly duration assigned to surgical specialty p to the median value of the weekly time used, in the last trimester, by the surgeons from the same surgical specialty with no individual assignment to OR time blocks, respectively.

3.3 Domain constraints

Finally, the variables domain is presented. The decision variables' domain is firstly stated. Then, the auxiliary variables' domain is settled by type: binary, positive integer, and positive real.

- Decision variables

$$durb_{srdk} \geq 0 \text{ and integer}, \forall s \in S,\ r \in R,\ d \in A,\ k \in K \tag{3.38}$$

$$x_{srdk} \in \{0,1\}, \forall s \in S,\ r \in R,\ d \in A,\ k \in K \tag{3.39}$$

$$y_{prdk} \in \{0,1\}, \forall p \in P,\ r \in R,\ d \in A,\ k \in K \tag{3.40}$$

- Binary auxiliary variables

$$dev^{-}_{pdk},\ dev^{+}_{pdk} \in \{0,1\}, \forall p \in P,\ d \in A,\ k \in K \tag{3.41}$$

$$room_{pr} \in \{0,1\}, \forall p \in P,\ r \in R \tag{3.42}$$

$$z_{rdk} \in \{0,1\}, \forall r \in R,\ d \in A,\ k \in K \tag{3.43}$$

$$z^{day}_{sd} \in \{0,1\}, \forall s \in S,\ d \in A \tag{3.44}$$

$$z^{week}_{s} \in \{0,1\}, \forall s \in S \tag{3.45}$$

- Positive integer auxiliary variables

$$dev^{SPC}_{p-},\ dev^{SPC}_{p+} \geq 0 \text{ and integer},\ \forall p \in P \tag{3.46}$$

$$dev^{SRG}_{s-},\ dev^{SRG}_{s+} \geq 0 \text{ and integer},\ \forall s \in S \tag{3.47}$$

$$Idev_{pdk} \geq 0 \text{ and integer}, \forall p \in P,\ d \in A,\ k \in K \tag{3.48}$$

$$Max_{p} \geq 0 \text{ and integer}, \forall p \in P \tag{3.49}$$

$$u_{srdk} \geq 0 \text{ and integer}, \forall s \in S,\ r \in R,\ d \in A,\ k \in K \tag{3.50}$$

- Positive real auxiliary variables

$$\overline{ad_h},\, \overline{mean_h} \geq 0,\, \forall h \in H \tag{3.51}$$

$$ad_{dh},\, mean_{dh} \geq 0,\, d \in A,\, h \in H \tag{3.52}$$

$$d^{SPC}_{pdhq} \geq 0,\, \forall p \in P,\, d \in A,\, h \in H,\, q \in Q^{SPC}_p \tag{3.53}$$

$$d^{SRG}_{sdhq} \geq 0,\, \forall s \in S,\, d \in A,\, h \in H,\, q \in Q^{SRG}_s \tag{3.54}$$

$$m^{SPC}_{pdh} \geq 0,\, \forall p \in P,\, d \in A,\, h \in H \tag{3.55}$$

$$m^{SRG}_{sdh} \geq 0,\, \forall s \in S,\, d \in A,\, h \in H \tag{3.56}$$

$$n^{SPC}_{pdq} \geq 0,\, \forall p \in P,\, d \in A,\, q \in Q^{SPC}_p \tag{3.57}$$

$$n^{SRG}_{sdq} \geq 0,\, \forall s \in S,\, d \in A,\, q \in Q^{SRG}_s \tag{3.58}$$

Chapter 4

Model implementation

This chapter discusses the complexity of the model in terms of the number of variables and constraints, and how the complexity increases when the number of surgeons, of active days, and of operating rooms increases. Then the implementation process will be detailed, focusing on the data processing and parameters' analysis. The chapter ends with a short description of the instances that will be used in the next chapter for model testing and corresponding solution analysis.

4.1 Complexity of the model

Consider that $|A|$, $|H|$, $|K|$, $|P|$, $|Q_p^{SPC}|$, $|Q_s^{SRG}|$, $|R|$, $|S|$ represent the number of elements of the corresponding sets. Table 4.1 shows the number of (decision and auxiliary) variables, while Table 4.2 summarizes the number of constraints of the mixed-integer linear programming (MILP) model presented in the previous chapter.

It was possible to create three instances to test the modelling approach from the hospital records (OR historical data from 2013 and 2014). As said before, the hospital OR has been growing mainly because the number of surgeons and surgeries has been increasing accordingly. For the most recent instance, which has the highest number of surgeons, the arising MILP problems have approximately 77000 constraints (excluding domain constraints), and about 75000 variables. Table 4.1 and Table 4.2 also present the approximated number of variables and constraints, respectively, of the most recent instance of the MILP model. The numbers are presented in approximated values as the number of classes of the variables dur_{sq}^{SRG} and dur_{pq}^{SPC} vary with the surgeon or surgical specialty under analysis, and so a mean value of six and twelve, respectively, is defined in order to perform the calculations.

Over the three instances, the number of active days, hospitalization units, shifts, surgical

Table 4.1: Number of variables

Type	Number	Example (3rd instance)
$durb_{srdk}$	$\lvert S\rvert \cdot \lvert R\rvert \cdot \lvert A\rvert \cdot \lvert K\rvert$	15680
x_{srdk}	$\lvert S\rvert \cdot \lvert R\rvert \cdot \lvert A\rvert \cdot \lvert K\rvert$	15680
y_{prdk}	$\lvert P\rvert \cdot \lvert R\rvert \cdot \lvert A\rvert \cdot \lvert K\rvert$	910
$\overline{ad_h}$	$\lvert H\rvert$	2
ad_{dh}	$\lvert A\rvert \cdot \lvert H\rvert$	10
d^{SPC}_{pdhq}	$\lvert P\rvert \cdot \lvert A\rvert \cdot \lvert H\rvert \cdot \lvert Q^{SPC}_p\rvert$	1560
d^{SRG}_{sdhq}	$\lvert S\rvert \cdot \lvert A\rvert \cdot \lvert H\rvert \cdot \lvert Q^{SRG}_s\rvert$	13440
dev^-_{pdk}, dev^+_{pdk}	$2 \cdot \lvert P\rvert \cdot \lvert A\rvert \cdot \lvert K\rvert$	260
dev^{SPC}_{p-}, dev^{SPC}_{p+}	$2 \cdot \lvert P\rvert$	26
dev^{SRG}_{s-}, dev^{SRG}_{s+}	$2 \cdot \lvert S\rvert$	448
$Idev_{pdk}$	$\lvert P\rvert \cdot \lvert A\rvert \cdot \lvert K\rvert$	130
Max_p	$\lvert P\rvert$	13
m^{SPC}_{pdh}	$\lvert P\rvert \cdot \lvert A\rvert \cdot \lvert H\rvert$	130
m^{SRG}_{sdh}	$\lvert S\rvert \cdot \lvert A\rvert \cdot \lvert H\rvert$	2240
$\overline{mean_h}$	$\lvert H\rvert$	2
$mean_{dh}$	$\lvert A\rvert \cdot \lvert H\rvert$	10
n^{SPC}_{pdq}	$\lvert P\rvert \cdot \lvert A\rvert \cdot \lvert Q^{SPC}_p\rvert$	780
n^{SRG}_{sdq}	$\lvert S\rvert \cdot \lvert A\rvert \cdot \lvert Q^{SRG}_s\rvert$	6720
$room_{pr}$	$\lvert P\rvert \cdot \lvert R\rvert$	91
u_{srdk}	$\lvert S\rvert \cdot \lvert R\rvert \cdot \lvert A\rvert \cdot \lvert K\rvert$	15680
z_{rdk}	$\lvert R\rvert \cdot \lvert A\rvert \cdot \lvert K\rvert$	70
z^{day}_{sd}	$\lvert S\rvert \cdot \lvert A\rvert$	1120
z^{week}_s	$\lvert S\rvert$	224
Total		75226

specialties, and rooms remain the same, while the number of surgeons increases. With the hospital physical expansion, and the continuing increase of the workflow, the most likely to be incremented is the number of active days, rooms, and surgeons to be considered. By incrementing in one unit the number of surgeons, the arising MILP problems have about more 300 constraints (excluding domain constraints), and 300 variables. By incrementing in one unit the number of rooms, the arising MILP problems have approximately more 4500 constraints, and 3500 variables, while by incrementing in one unit the number of active days, the arising MILP problems have about more 15000 constraints, and 15000 variables.

It is expected that the MILP formulation will not be easy to compute, since it has many conflictual terms in the objective function, and it will run with real instances from a medium size hospital that is growing and probably will continue to grow fast.

4.2 Model implementation

The collected data shows, for each surgery performed in 2013 and 2014, some information about the patient: the patient number (in the hospital IT system), the patient gender, and the patient date of birth. It also shows some surgery information: the episode or surgery number (in the hospital IT system), the surgical proposal date, the surgery date, the time at which the patient was admitted and left the OR, the room, and the PACU, and the start and end time of the patient anesthesia and surgery. The surgical proposal date is inaccurate for most surgeries, and therefore impossible to be properly used. The surgical specialty, the type of anesthesia, and the procedures code and description are also presented. Note that a surgery tends to have more than one surgical procedure. The data includes a field where surgeries are classified as outpatient or inpatient, and a field where surgeries are classified as elective or urgent. The data also includes a number associated to each member of the surgical team (in the hospital IT system): surgeon, assistant surgeons, OR technician, and anesthetist. The patient number, and the number of each member of the surgical team were codified in order not to expose confidential information. A surgery does not need to have assigned all the described members of the surgical team. It can, for example, have two assistant surgeons, or none OR technician.

This information was given in an Excel file, that was converted to a text file, in order to import the information to R, a free software environment for statistical computing and graphics (*The R Project for Statistical Computing* 2015). This software was chosen to handle the data received: to perform some data consistency analysis, and to extract the necessary inputs to the MILP model in the desired format.

Besides the numbers and statistics presented throughout Chapter 2, there are other relevant outputs from the referred analysis.

- On average one in every seven patients performs two surgeries per year, while others perform just one.
- Elective surgeries were held on more than one third of the weekend days (Saturdays and Sundays) or national holidays. This fact led to a discussion with the head doctor of the surgical suite, on which he admitted that the OR opened in some Saturdays, in order to overcome the lack of OR availability during the regular opening hours. He also claims that

opening the OR on Saturdays, in part or full time, is an option (if the workload justifies so).

- About one third of all surgeries are ambulatory surgeries, which means that approximately two thirds of the operated patients are being sent to the ICU and wards.
- By crossing the surgeon information with the performed surgical procedures, and the surgical specialty of the surgeries, it was detected that some surgeries had the surgical specialty code mistyped. After a case by case analysis, it was possible to correct the data manually.
- Six different types of anesthesia were performed, and some of the surgeries have not this field filled in the IT system. This information could be recovered in most cases, however this information will not be considered in this study.
- About 890 different surgical procedures were executed, and some of the surgeries have not this field filled in the IT system. It should not be possible to select options as "procedure to be defined", since it creates opportunity for people not to answer properly, making the data less valuable. It will be almost impossible to recover this type of information.
- Only about one third of the surgical procedures were performed more than five times per year, which will probably make it impossible to accurately estimate the duration of the surgeries.
- Only less than a quarter of the surgeons performed on average more than one surgery per day, so it is possible to conclude that a large portion of surgeons will not obtain an individual surgical time in the MSS.
- In 2013 and 2014, there were 25 and 28 active anesthesiologists, respectively. However, in 2013, almost a third of the surgeries have not an assigned anesthesiologist. This problem was corrected in 2014, since only 3% of the performed surgeries have not an assigned anesthesiologist. It is important to motivate the staff to understand the need of correctly filling the surgery form.
- Despite the unfilled or improper filled fields, after the analysis and manual changes, less than ten surgeries were excluded from the study.
- The peak number of surgeries occurs in the middle of the week, so it could be interesting to study some intra-week seasonality, and also the effect of the artificial variability on the quality of care, and on the quality of the work and satisfaction of the hospital staff. However, this is not the scope of this work.
- July and August are the months with less surgeries (see weeks 29 to 37, and 82 to 88 in Figure 2.1), followed by a peak of surgeries in the winter season, until Christmas and new

year holidays where the number of surgeries decreases abruptly (weeks 1, 52, 53, 104, and 105 in Figure 2.1). It could also be interesting to study some yearly seasonality.

- The median number of surgeries daily performed increased from 28 surgeries in 2013, to 30 surgeries in 2014. A higher rise is expected in the following year.

A new MSS was implemented in the beginning of 2015, and the MSS needs to be reviewed in periods of about three months. In order to compare the MILP model results (and to adjust the model parameters), three instances were created from the historical data. The most recent instance aims to create the MSS for the first quarter of 2015, and its results will be compared with the actual MSS implemented in this period. This instance uses the last three months of available data, from October to December of 2014, to estimate the inputs needed to the MILP model. However, when the surgeon (or surgical specialty) under study has no surgeries in the last trimester, the data related to the surgeries performed by that surgeon (or surgical specialty) in the previous year (entire 2014) are analyzed instead. Note that the median value of the weekly time used by this type of surgeons, who do not have any surgeries performed in the last trimester, is expected to be lower, and so it is expected that the surgeons are not individually assigned in the MSS. The inputs concerning the surgeries duration, and the probability for a patient to be sent to each hospitalization unit per surgeon or surgical specialty are an exception, since they are calculated using all historical available data. Thus it is expected that the values obtained are a better estimation of the real values.

The second and third instances were created following the same procedure. The second instance uses the trimester from July to September of 2014, and when no surgery is found for a specific surgeon (or surgical specialty), it analyses the data from October of 2013 to September of 2014. It aims to create the MSS for the fourth trimester of 2014. The third instance is based on the second trimester of 2014, April to June of 2014. When needed (based on the same reasoning), it searches surgeries from July of 2013 to June of 2014, and it pretends to establishes the MSS for July to September of 2014.

After being extracted all the inputs needed for the MILP, R printed them in text files. Some changes were manually executed in those files in order to make them readable by the Mosel language (Colombani & Heipcke 2002), which was used to implement the described formulation, and to create the corresponding LP files. The MILP model was optimized on DOcplexcloud, the Decision Optimization on Cloud, with a machine that possesses ten cores and 60 gigabytes of memory. On the IBM CPLEX optimizer on Cloud, LP files are executed by the order of submission, one at a time, with a time limit of 60 minutes (*IBM Decision Optimization on Cloud* 2016).

4.3 Instances description

The three instances have some values in common, due to the problem description. Since the surgical suite regular opening hours is from 8am to 11pm, from Monday to Friday, the implemented mathematical model consider $A = \{1, 2, 3, 4, 5\}$, where e.g. $A = 1$ corresponds to Monday and $A = 5$ corresponds to Friday. Even though some surgeries have been carried out on Saturday, the OR is not open on Saturdays on a regular basis, and so it will not be considered as an active day. However, if this work further concludes that more available OR time is needed, Saturday may then be considered as an active day.

As outlined in figures 2.4 and 2.5, the patients can go through four care units. The PACU does not suffer from major fluctuations since the length of stay is usually short and does not differ significantly. The number of patients at ICU is also hard to predict since the length of stay is highly variable. In the hospital historical data, there is no information concerning the presence of the patient in the ICU. For this reason, only the outpatient care unit (ACU) and the inpatient care unit (ICU and wards) are considered in the tests of the MILP model. Given that, $H = \{1, 2\}$, where $h = 1$ corresponds to the outpatient care unit, and $h = 2$ corresponds to the inpatient care unit.

The surgical suite opening hours are divided in two shifts, so $K = \{1, 2\}$, where $k = 1$ corresponds to the morning shift, and $k = 2$ corresponds to the afternoon shift.

All the thirteen surgical specialties (presented in Figure 2.2) are valences of the hospital during the periods of the three instances. Therefore $P = \{1, \ldots, 13\}$, where e.g. $P = 5$ corresponds to the surgical specialty coded 5, Plastic Surgery. The average number of classes per surgical specialty (over the three instances) is twelve, and it varies in the range 6-18.

The surgical suite has seven operational rooms, thus $R = \{1, 2, 3, 4, 5, 6, 7\}$, where e.g. $R = 3$ corresponds to the room number three.

However, the number of surgeons has been increasing. The instance using the oldest historical data has 206 active surgeons. Accordingly, $S = \{1, \ldots, 206\}$, where e.g. $S = 9$ corresponds to the surgeon which ID number is 9. The second instance has 217 active surgeons, and the instance using the most recent historical data has 224 active surgeons. Thereafter, $S = \{1, \ldots, 217\}$ on the second instance, and $S = \{1, \ldots, 224\}$ on the instance with the newer data. The average number of classes per surgeon (over the three instances) decreases a few tenths from seven to six, and it varies in the range 1-15 (over all three instances).

Table 4.3 summarizes the presented information, and so the characteristics of the three instances, whose optimization results will be presented in the next chapter.

Table 4.2: Number of constraints

Type	Number	Example (3rd instance)
(3.1)	$\|S\| \cdot \|A\| \cdot \|K\|$	2240
(3.2)	$\|R\| \cdot \|A\| \cdot \|K\|$	70
(3.3)	$\|R\| \cdot \|A\| \cdot \|K\|$	70
(3.4)	$\|R\| \cdot \|A\| \cdot \|K\|$	70
(3.5)	$\|S\| \cdot \|R\| \cdot \|A\| \cdot \|K\|$	15680
(3.6)	$\|S\| \cdot \|R\| \cdot \|A\| \cdot \|K\|$	15680
(3.7)	$\|S\| \cdot \|A\|$	1120
(3.8)	$\|S\|$	224
(3.9)	$\|S\| \cdot \|A\|$	1120
(3.10)	$\|S\|$	224
(3.11)	1	1
(3.12)	1	1
(3.13)	$\|S\| \cdot \|A\| \cdot \|Q_s^{SRG}\|$	6720
(3.14)	$\|P\| \cdot \|A\| \cdot \|Q_p^{SPC}\|$	780
(3.15)	$\|S\| \cdot \|A\| \cdot \|H\|$	2240
(3.16)	$\|P\| \cdot \|A\| \cdot \|H\|$	130
(3.17), (3.18)	$2 \cdot \|S\| \cdot \|A\| \cdot \|H\| \cdot \|Q_s^{SRG}\|$	26880
(3.19), (3.20)	$2 \cdot \|P\| \cdot \|A\| \cdot \|H\| \cdot \|Q_p^{SPC}\|$	3120
(3.21)	$\|A\| \cdot \|H\|$	10
(3.22)	$\|A\| \cdot \|H\|$	10
(3.23)	$\|A\| \cdot \|H\|$	10
(3.24)	$\|A\| \cdot \|H\|$	10
(3.26)	$\|P\| \cdot \|R\|$	91
(3.29)	$\|P\| \cdot \|A\| \cdot \|K\|$	130
(3.30)	$\|P\| \cdot \|A\| \cdot \|K\|$	130
(3.31)	$\|P\| \cdot \|A\| \cdot \|K\|$	130
(3.33)	$\|P\|$	13
(3.34)	$\|P\|$	13
(3.35)	$\|S\|$	224
(3.36)	$\|P\|$	13
Total		77154

Table 4.3: Characteristics of the three instances

Set	Description	Instance	Cardinality
$\|A\|$	nb active days	1st, 2nd, 3rd	5
$\|H\|$	nb hospitalization units	1st, 2nd, 3rd	2
$\|K\|$	nb shifts	1st, 2nd, 3rd	2
$\|P\|$	nb surgical specialties	1st, 2nd, 3rd	13
$\|Q_p^{SPC}\|$	nb classes per surgical specialty	1st, 2nd, 3rd	6-18
$\|Q_s^{SRG}\|$	nb classes per surgeon	1st, 2nd, 3rd	1-15
$\|R\|$	nb OR	1st, 2nd, 3rd	7
$\|S\|$	nb surgeons	1st (newest)	224
$\|S\|$	nb surgeons	2nd	217
$\|S\|$	nb surgeons	3rd (oldest)	206

Chapter 5

Solution analysis

This chapter presents the results of the computational experiments performed to test the solution approach with real data from the hospital. Tests focused on the first instance described below (see Section 5.1). The remaining instances were also tested and led to similar results (see Section 5.2). Since the problem's objective function is formulated as a weighted sum of the multiple criteria presented, 35 algorithm runs were performed with different weight values, in order to get a good overview of the solution space. The results obtained are analyzed and compared with the MSS collected from the hospital (see Section 5.3). Since it is so difficult to objectively compare the quality of the different generated schedules, as there is no unique objective measure to make this comparison, the main goal of this chapter is to build a quality schedule or at least to improve the current schedule.

5.1 First instance results

Figure 5.1 shows, for each algorithm run (in columns), the relative importance assigned to each term of the objective function, the best bound found, the best solution objective value, the corresponding gap, the solution time (one-hour time limit is used), the algorithm run number, the value obtained for each term of the objective function, and the deviation between each term value and the corresponding best value found over all algorithm runs (in percentage). The last five columns present the minimum, maximum, mean, and median value of each parameter listed above, over all algorithm runs. It is also presented the number of times an optimal solution is found, and the difference between the maximum and the minimum value obtained for each term of the objective function, over all algorithm runs. A legend for the first column of the figure is presented on Table 5.1.

On the 35 algorithm runs using the first instance, the model is able to find 16 times (46%)

	Relative importance of each term of the objective function																			
1st OC: Wmean	1	0	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8
1st OC: Wad	0	1	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8
2nd OC: Wroom	0	0	1	0	0	0	0	0	0	0	0	0	0	1/4	1/5	1/2	1/3	1/4	1/4	1/4
3rd OC: Wdev	0	0	0	1	0	0	0	0	0	0	0	0	1/3	1/4	1/5	0	1/3	1/4	1/4	0
4th OC: Wdev^SRG_less	0	0	0	0	1	0	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8
4th OC: Wdev^SRG_more	0	0	0	0	0	1	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8
4th OC: Wdev^SPC_less	0	0	0	0	0	0	1	0	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8
4th OC: Wdev^SPC_more	0	0	0	0	0	0	0	1	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8
	Best solution found																			
Best bound	39,3999	9,2492	6	0	0	10561	0	0	50,2548	10561	0	12121	12121	12129,6552	12199,6851	57,9252	57,7680	10638,4178	68,2585	12199,8593
Objective value	39,5024	9,4556	6	0	0	10561	0	0	50,9473	10561	0	12121	12121	12134	12210,3466	68,2456	66,6882	10658,0883	87,8398	12209,9917
Gap	0,26%	2,18%	0%	0%	0%	0%	0%	0%	1,36%	0%	0%	0%	0%	0,04%	0,09%	15,12%	13,38%	0,18%	22,29%	0,08%
Solution time	3600s	3600s	267.62s	2.46s	2.92s	8.74s	3.17s	3.48s	3600s	4.45s	6.13s	9.63s	11.53s	3600s	3600s	3600s	3600s	3600s	3600s	3600s
Run number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	Obtained value for each term of the objective function																			
1st OC: mean	39,5024	56	76	79	96	85	108	83	40,3222	90	79	92	99	78	54,1226	41,5926	41,9965	54,9802	49,5839	52,9664
1st OC: ad	41	9,4556	58	61	71	70	78	71	10,6251	61	66	72	69	65	17,2240	11,6530	11,6916	18,1081	15,2560	17,0253
2nd OC: room	91	91	6	91	91	91	91	91	91	91	91	91	91	13	18	15	13	23	23	19
3rd OC: dev_more	2	1	4	0	0	4	7	0	1	3	1	1	0	0	0	2	0	1	0	0
4th OC: dev^SRG_less	8656	9453	7333	7638	0	0	5327	7794	8596	0	4332	0	0	0	0	8100	7679	0	4655	0
4th OC: dev^SRG_more	19877	20734	18134	19699	14521	10561	17268	19135	20237	10561	16873	11641	12121	11221	12121	18961	19380	10561	16896	12121
4th OC: dev^SPC_less	8336	8911	5813	5311	0	0	0	5058	8549	0	0	0	0	0	0	6963	7434	0	0	0
4th OC: dev^SPC_more	831	841	1716,5	313	1740	1860	3339	0	481	2100	0	480	0	900	0	727	481	1920	0	0
	Deviation between each term value and the corrresponding best value found																			
1st OC: mean	0%	29,46%	48,02%	50,00%	58,85%	53,53%	63,42%	52,41%	2,03%	56,11%	50,00%	57,06%	60,10%	49,36%	27,01%	5,03%	5,94%	28,15%	20,33%	25,42%
1st OC: ad	76,94%	0%	83,70%	84,50%	86,68%	86,49%	87,88%	86,68%	11,01%	84,50%	85,67%	86,87%	86,30%	85,45%	45,10%	18,86%	19,13%	47,78%	38,02%	44,46%
2nd OC: room	93,41%	93,41%	0%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	53,85%	66,67%	60,00%	53,85%	73,91%	73,91%	68,42%
3rd OC: dev_more	100%	100%	100%	0%	0%	100%	100%	0%	100%	100%	100%	100%	0%	0%	0%	100%	0%	100%	0%	0%
4th OC: dev^SRG_less	100%	100%	100%	100%	0%	0%	100%	100%	100%	0%	100%	0%	0%	0%	0%	100%	100%	0%	100%	0%
4th OC: dev^SRG_more	46,87%	49,06%	41,76%	46,39%	27,27%	0%	38,84%	44,81%	47,81%	0%	37,41%	9,28%	12,87%	5,88%	12,87%	44,30%	45,51%	0%	37,49%	12,87%
4th OC: dev^SPC_less	100%	100%	100%	100%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0%	0%	100%	100%	0%	0%	0%
4th OC: dev^SPC_more	100%	100%	100%	100%	100%	100%	100%	0%	100%	100%	0%	100%	0%	100%	0%	100%	100%	100%	0%	0%

	Relative importance of each term of the objective function																			
1st OC: Wmean	1/8	1/5	0	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10					
1st OC: Wad	1/8	0	1/5	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10					
2nd OC: Wroom	0	1/5	1/5	1/5	1/5	1/5	1/5	0	0	0	0	1/4	1/4	1/5	1/5					
3rd OC: Wdev	1/4	1/5	1/5	1/5	1/5	1/5	1/5	0	0	1/3	1/3	1/4	1/4	1/5	1/5					
4th OC: Wdev^SRG_less	1/8	1/10	1/10	0	1/5	1/10	1/10	1/2	0	1/3	0	1/4	0	1/5	0					
4th OC: Wdev^SRG_more	1/8	1/10	1/10	1/5	0	1/10	1/10	0	1/2	0	1/3	0	1/4	0	1/5					
4th OC: Wdev^SPC_less	1/8	1/10	1/10	1/10	1/10	0	1/5	1/2	0	1/3	0	1/4	0	1/5	0					
4th OC: Wdev^SPC_more	1/8	1/10	1/10	1/10	1/10	1/5	0	0	1/2	0	1/3	0	1/4	0	1/5					
	Best solution found															MIN	MAX	MEAN	MEDIAN	Number of optimal solutions
Best bound	12190,0621	12181,2349	12144,4527	12195,5957	79,8273	12198,2491	10638,8298	0	12121	0	12121	13	12128,7578	80,0986	12194,8409	0	12199,9	6088,7835	10561	
Objective value	12190,0621	12187,8268	12152,9652	12217,2239	88,3962	12212,1710	10655,4535	0	12121	0	12121	13	12134	99,2894	12221,4370	0	12221	6094,8266	10561	
Gap	0%	0,05%	0,07%	0,18%	9,69%	0,11%	0,16%	0%	0%	0%	0%	0%	0,04%	19,33%	0,22%	0%	22,29%	2,42%	0,04%	16
Solution time	663.11s	3600s	3600s	3600s	3600s	3600s	3600s	2.32s	4.75s	3.51s	6.13s	234.21s	3600s	3600s	3600s					
Run number	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35					
	Obtained value for each term of the objective function															MIN	MAX	MEAN	MEDIAN	DIFFERENCE
1st OC: mean	52,5045	51,8268	65	53,3444	54,0352	53,5634	55,2530	89	90	86	79	88	73	56,0379	52,4732	39,5024	108	68,4316	65	68,4976
1st OC: ad	16,5576	56	16,9652	16,8796	17,3609	17,6076	18,2004	74	70	68	67	69	59	18,2515	16,9638	9,4556	78	42,7379	56	68,5444
2nd OC: room	91	15	15	24	17	20	21	91	91	91	91	13	13	25	30	6	91	53,4286	30	85
3rd OC: dev_more	0	0	0	2	0	0	0	2	3	0	0	0	0	0	1	0	7	1	0	7
4th OC: dev^SRG_less	0	0	0	1358	0	0	0	0	1079	0	954	0	847	0	1283	0	9453	2430,9714	0	9453
4th OC: dev^SRG_more	11641	12121	12121	12099	12181	12121	10561	13441	11820	13501	11635	10801	11588	12181	12084	10561	20734	14071,9714	12121	10173
4th OC: dev^SPC_less	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8911	1610,7143	0	8911
4th OC: dev^SPC_more	480	0	0	22	0	0	1740	1800	301	480	486	1380	533	0	37	0	3339	713,9571	480	3339
	Deviation between each term value and the corrresponding best value found															MIN	MAX	MEAN	MEDIAN	
1st OC: mean	24,76%	23,78%	39,23%	25,95%	26,90%	26,25%	28,51%	55,62%	56,11%	54,07%	50,00%	55,11%	45,89%	29,51%	24,72%	0%	63,42%	37,39%	39,23%	
1st OC: ad	42,89%	83,12%	44,27%	43,98%	45,54%	46,30%	48,05%	87,22%	86,49%	86,09%	85,89%	86,30%	83,97%	48,19%	44,26%	0%	87,88%	63,10%	83,12%	
2nd OC: room	93,41%	60,00%	60,00%	75,00%	64,71%	70,00%	71,43%	93,41%	93,41%	93,41%	93,41%	53,85%	53,85%	76,00%	80,00%	0%	93,41%	77,24%	80,00%	
3rd OC: dev_more	0%	0%	0%	100%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0%	100%	0%	100%	42,86%	0%	
4th OC: dev^SRG_less	0%	0%	0%	100%	0%	0%	0%	0%	100%	0%	100%	0%	100%	0%	100%	0%	100%	45,71%	0%	
4th OC: dev^SRG_more	9,28%	12,87%	12,87%	12,71%	13,30%	12,87%	0%	21,43%	10,65%	21,78%	9,23%	2,22%	8,86%	13,30%	12,60%	0%	49,06%	21,01%	12,87%	
4th OC: dev^SPC_less	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%	22,86%	0%	
4th OC: dev^SPC_more	100%	0%	0%	100%	0%	0%	100%	100%	100%	100%	100%	100%	100%	0%	100%	0%	100%	68,57%	100%	

Figure 5.1: Solutions obtained for the first instance, after 35 algorithm runs

an optimal solution in less than one hour of computation (77s on average). On an overall perspective, the model obtained the worst gap of 22.29% (see run 19), an average gap of 2.42% and a median gap of 0.04%, within a computing time limit of one hour.

By analyzing the results obtained from the various algorithm runs, one can conclude that

Table 5.1: Legend of the first column of Figures 5.1 to 5.6

Text nomenclature	**Figure nomenclature**
Optimization Criterion	OC
$\sum_{h \in H} Wmean_h$	Wmean
$\sum_{h \in H} Wad_h$	Wad
$\sum_{p \in P} Wroom_p$	Wroom
$\sum_{p \in P} Wdev_p$	Wdev
$\sum_{s \in S} Wdev_{s-}^{SRG}$	Wdev^SRG_less
$\sum_{s \in S} Wdev_{s+}^{SRG}$	Wdev^SRG_more
$\sum_{p \in P} Wdev_{p-}^{SPC}$	Wdev^SPC_less
$\sum_{p \in P} Wdev_{p+}^{SPC}$	Wdev^SPC_more
$\sum_{h \in H} \overline{mean_h}$	mean
$\sum_{h \in H} \overline{ad_h}$	ad
$\sum_{p \in P} \sum_{r \in R} room_{pr}$	room
$\sum_{p \in P} \sum_{d \in A} \sum_{k \in K} dev_{pdk}^{+}$	dev_more
$\sum_{s \in S} dev_{s-}^{SRG}$	dev^SRG_less
$\sum_{s \in S} dev_{s+}^{SRG}$	dev^SRG_more
$\sum_{p \in P} dev_{p-}^{SPC}$	dev^SPC_less
$\sum_{p \in P} dev_{p+}^{SPC}$	dev^SPC_more

the model can easily find the solution when the terms of the second, third and fourth criteria are considered (have relative importance greater than zero) individually or together (see runs 3-8, 10-14, and 28-32). However, when the first criterion is considered, separated or together with at least one of the remaining criteria, the model shows difficulties in finding an optimal solution (see runs 1-2, 9, 15-20, 22-27, and 33-35). An exception is run 21 that found an optimal solution even considering all the criteria excluding the second one. The worst gaps (greater than 5%) occur five times (see runs 16, 17, 19, 25, and 34), when optimizing at least the first two criteria without giving weight to $\sum_{s \in S} dev_{s+}^{SRG}$, what makes it possible to conclude that this term of the forth criterion is of high importance on the optimization process. Remember that the corresponding parameters, $Wdev_{s+}^{SRG}$, give the relative importance of diverging positively the weekly duration assigned to each surgeon to the corresponding median value of the weekly time used in the last trimester.

Special remark for run 15 that considers all terms of all four criteria and obtained an overall gap of 0.09%, which is quite good. This run obtained a solution that has a good deviation for the third and the forth criterion, but neglect the first and second criteria. This may be happening because the four optimization criteria of the objective function do not vary in the same order of magnitude. The first three terms vary from units to hundreds, while the last criterion can be equal to thousands of units. Given that, the analysis proceeds with Figure 5.2 that shows the results obtained after normalizing by the ratio of differences the objective function. This normalization method divides the absolute difference that each algorithm run presents in each criterion term relative to the worst performance of the corresponding criterion term, by the difference between the best and worst performance of this criterion term. The normalization by ratio of differences do not preserve the ratios between algorithm runs, but assures the use of the full range of values. The best and worst performance of each criterion term were estimated by the best and worst value obtained over all the 35 algorithm runs, respectively. Alternatively, one could use the normalization by the ratio, but it does not fully solve the problem, since the first, third and fourth term of the forth criterion vary from 0 to more than 3000, while the second varies from 10561 to 20734, and so the result of dividing each algorithm run value by the best performance on the corresponding criterion will not work as desirable.

After performing the normalization of the objective function, the model is able to find 15 times (43%) an optimal solution in less than one hour of computation (252s on average). Unlike the initial model, this normalized model cannot find an optimal solution for run 21, and it cannot conclude that the solution found for run 32 is an optimal one. On an overall perspective, the normalized model obtained the worst gap of 82.14% (see run 34), an average gap of 9.16% and a median gap of 0.18%, within a computing time limit of one hour. Taking this into account, the average time needed to find an optimal solution increased significantly as it was expected, since

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10	Run 11	Run 12	Run 13	Run 14	Run 15	Run 16	Run 17	Run 18	Run 19	Run 20	Run 21
Relative importance of each term of the objective function																					
1st OC: Wmean	1	0	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8	1/8
1st OC: Wad	0	1	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8	1/8
2nd OC: Wroom	0	0	1	0	0	0	0	0	0	0	0	0	0	1/4	1/5	1/2	1/3	1/4	1/4	1/4	0
3rd OC: Wdev	0	0	0	1	0	0	0	0	0	0	0	0	1/3	1/4	1/5	0	1/3	1/4	1/4	0	1/4
4th OC: Wdev^SRG_less	0	0	0	0	1	0	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8	1/8
4th OC: Wdev^SRG_more	0	0	0	0	0	1	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8	1/8
4th OC: Wdev^SPC_less	0	0	0	0	0	0	1	0	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8	1/8
4th OC: Wdev^SPC_more	0	0	0	0	0	0	0	1	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8	1/8
Best solution found																					
Best bound	39,3999	9,2492	6	0	0	10561	0	0	50,2548	10561	0	12121	12121	12129,6552	12199,6851	57,9252	57,7680	10638,4178	68,2585	12199,8593	12190,0621
Objective value	39,5124	9,6256	6	0	0	10561	0	0	51,5539	10561	0	12121	12121	12134	12258,1176	66,9363	67,6807	11133,4343	288,3230	14090,4098	12213,7612
Gap	0,28%	3,91%	0%	0%	0%	0%	0%	0%	2,52%	0%	0%	0%	0%	0,04%	0,48%	13,46%	14,65%	4,45%	76,33%	13,42%	0,19%
Solution time	3600s	3600s	59.54s	3.20s	3.15s	6.74s	3.56s	0.45s	3600s	5.16s	10.25s	9.48s	11.04s	3600s	3600s	3600s	3600s	3600s	3600s	3600s	3600s
Run number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Obtained value for each term of the objective function																					
1st OC: mean	39,5124	52	73	86	87	89	91	115	40,7835	87	91	88	88	78	54,9567	40,9580	42,3266	56,6110	47,3297	57,7237	53,0840
1st OC: ad	44	9,6256	54	69	74	69	71	77	10,7704	70	76	78	66	62	18,1610	10,9783	11,3542	18,8233	13,4933	18,6861	16,6772
2nd OC: room	91	91	6	91	91	91	91	91	91	91	91	91	91	13	23	15	14	25	24	34	91
3rd OC: dev_more	1	2	2	0	1	1	9	0	1	3	0	0	0	0	0	0	0	0	0	3	0
4th OC: dev^SRG_less	8383	9133	6496	5668	0	0	5645	6257	8560	0	3260	0	0	0	4	8054	7310	236	5720	1521,5	11,5
4th OC: dev^SRG_more	19544	20294	17297	16949	15361	10561	16446	24258	20201	10561	18921	12121	12121	12121	12185	19755	18531	10797	17061	12322,5	12132,5
4th OC: dev^SPC_less	8213	8394	5773	2914	0	0	0	3139	6983	0	0	0	0	0	0	7278	6876	168	179,5	115	0
4th OC: dev^SPC_more	891	961	1240,5	481	840	1740	1964	0	481	2220	0	0	0	0	0	481	901,5	1800,5	24	21	0
Deviation between each term value and the corrresponding best value found																					
1st OC: mean	0%	24,03%	45,89%	54,07%	54,59%	55,62%	56,59%	65,65%	3,14%	54,59%	56,59%	55,11%	55,11%	49,36%	28,12%	3,55%	6,67%	30,22%	16,54%	31,57%	25,59%
1st OC: ad	78,51%	1,77%	82,49%	86,30%	87,22%	86,30%	86,68%	87,72%	12,21%	86,49%	87,56%	87,88%	85,67%	84,75%	47,93%	13,87%	16,72%	49,77%	29,92%	49,40%	43,30%
2nd OC: room	93,41%	93,41%	0%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	53,85%	73,91%	60,00%	57,14%	76,00%	75,00%	82,35%	93,41%
3rd OC: dev_more	100%	100%	100%	0%	100%	100%	100%	0%	100%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%	0%
4th OC: dev^SRG_less	100%	100%	100%	100%	0%	0%	100%	100%	100%	0%	100%	0%	0%	0%	100%	100%	100%	100%	100%	100%	100%
4th OC: dev^SRG_more	45,96%	47,96%	38,94%	37,69%	31,25%	0%	35,78%	56,46%	47,72%	0%	44,18%	12,87%	12,87%	12,87%	13,33%	46,54%	43,01%	2,19%	38,10%	14,29%	12,95%
4th OC: dev^SPC_less	100%	100%	100%	100%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0%	0%	100%	100%	100%	100%	100%	0%
4th OC: dev^SPC_more	100%	100%	100%	100%	100%	100%	100%	0%	100%	100%	0%	0%	0%	0%	0%	100%	100%	100%	100%	100%	0%

	Run 22	Run 23	Run 24	Run 25	Run 26	Run 27	Run 28	Run 29	Run 30	Run 31	Run 32	Run 33	Run 34	Run 35					
Relative importance of each term of the objective function																			
1st OC: Wmean	1/5	0	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10					
1st OC: Wad	0	1/5	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10					
2nd OC: Wroom	1/5	1/5	1/5	1/5	1/5	1/5	0	0	0	0	1/4	1/4	1/5	1/5					
3rd OC: Wdev	1/5	1/5	1/5	1/5	1/5	1/5	0	0	1/3	1/3	1/4	1/4	1/5	1/5					
4th OC: Wdev^SRG_less	1/10	1/10	0	1/5	1/10	1/10	1/2	0	1/3	0	1/4	0	1/5	0					
4th OC: Wdev^SRG_more	1/10	1/10	1/5	0	1/10	1/10	0	1/2	0	1/3	0	1/4	0	1/5					
4th OC: Wdev^SPC_less	1/10	1/10	1/10	1/10	0	1/5	1/2	0	1/3	0	1/4	0	1/5	0					
4th OC: Wdev^SPC_more	1/10	1/10	1/10	1/10	1/5	0	0	1/2	0	1/3	0	1/4	0	1/5					
Best solution found															MIN	MAX	MEAN	MEDIAN	Number of optimal solutions
Best bound	12181,2349	12144,4527	12195,5957	79,8273	12198,2491	10638,8298	0	12121	0	12121	13	12128,7578	80,0986	12194,8409	0	12199,8593	6088,7835	10561	
Objective value	12203,5977	12156,6704	13071,5271	316,9879	14005,1299	11257,3129	0	12121	0	12121	13	12386	448,4345	13060,1874	0	14090	6310,9772	10561	
Gap	0,18%	0,10%	6,70%	74,82%	12,90%	5,49%	0%	0%	0%	0%	0%	2,08%	82,14%	6,63%	0%	82,14%	9,16%	0,18%	15
Solution time	3600s	3600s	3600s	3600s	3600s	3600s	2.87s	59.12s	1.59s	10.66s	3600s	3600s	3600s	3600s					
Run number	22	23	24	25	26	27	28	29	30	31	32	33	34	35					
Obtained value for each term of the objective function															MIN	MAX	MEAN	MEDIAN	DIFFERENCE
1st OC: mean	53	70	55,7263	53,1168	58,5329	58,3166	85	90	82	89	77	87	55,0421	57,3848	39,5124	115	67,2259	58,3166	75,4876
1st OC: ad	57	16,6704	17,8009	17,3711	19,5970	18,9963	79	67	76	71	66	64	18,3924	18,8026	9,6256	79	40,7806	19,5970	69,3744
2nd OC: room	19	19	47	21	33	36	91	91	91	91	13	10	23	33	6	91	54,8667	41,5	85
3rd OC: dev_more	0	0	0	0	0	0	4	1	0	0	0	0	0	0	0	9	0,9333	0	9
4th OC: dev^SRG_less	5,5	0	1470,5	225,5	923	261,5	0	420	0	0	0	255	284	2150	0	9133	2633	352	9133
4th OC: dev^SRG_more	12126,5	12121	12811,5	12346,5	12744	10882,5	14821	12121	14881	12121	10861	12376	11445	12951	10561	24258	14479,5161	12744	13697
4th OC: dev^SPC_less	0	0	118,5	0	0	0	0	0	0	0	0	255	68	606,5	0	8394	1702,6833	34	8394
4th OC: dev^SPC_more	0	0	21	0	227	1772	2220	0	0	0	1380	0	960	0	0	2220	665,3710	481	2220
Deviation between each term value and the corrresponding best value found															MIN	MAX	MEAN	MEDIAN	
1st OC: mean	24,90%	43,57%	29,11%	25,63%	32,51%	32,26%	53,53%	56,11%	51,83%	55,62%	48,70%	54,59%	28,23%	31,16%	0%	65,65%	38,30%	43,57%	
1st OC: ad	83,41%	43,28%	46,88%	45,57%	51,75%	50,22%	88,03%	85,89%	87,56%	86,68%	85,67%	85,23%	48,59%	49,71%	1,77%	88,03%	63,74%	82,49%	
2nd OC: room	68,42%	68,42%	87,23%	71,43%	81,82%	83,33%	93,41%	93,41%	93,41%	93,41%	53,85%	40,00%	73,91%	81,82%	0%	93,41%	79,33%	87,23%	
3rd OC: dev_more	0%	0%	0%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0%	0%	0%	100%	31,43%	0%	
4th OC: dev^SRG_less	100%	0%	100%	100%	100%	100%	0%	100%	0%	0%	0%	100%	100%	100%	0%	100%	68,57%	100%	
4th OC: dev^SRG_more	12,91%	12,87%	17,57%	14,46%	17,13%	3%	28,74%	12,87%	29,03%	12,87%	2,76%	14,67%	7,72%	18,45%	0%	56,46%	22,86%	14,67%	
4th OC: dev^SPC_less	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	100%	100%	100%	0%	100%	42,86%	0%	
4th OC: dev^SPC_more	0%	0%	100%	0%	100%	100%	100%	0%	0%	0%	100%	0%	100%	0%	0%	100%	57,14%	100%	

Figure 5.2: Solutions obtained for the first instance, after 35 algorithm runs of the normalized objective function

the normalization of the objective function "reduces the distance between different solutions", making it harder to the model searching and evaluating the neighborhood solutions.

By analyzing the results obtained from the various algorithm runs, one can conclude that the normalized model can also easily find the solution when the terms of the second, third and fourth criteria are considered (have relative importance greater than zero) individually or together (see runs 3-8, 10-14, and 28-32). An exception is run 33, which the model shows difficulties in finding an optimal solution (obtained a gap of 2.08%). As concluded with the initial model, when the first criterion is considered, separated or together with at least one of the remaining criteria, the normalized model also shows difficulties in finding an optimal solution (see runs 1-2, 9, 15-27, and 34-35). The worst gaps (greater than 70%) occur three times, when optimizing at least the first three criteria and one term of the forth criterion, without giving weight to $\sum_{s\in S} dev_{s+}^{SRG}$ (see runs 19, 25, and 34). The remaining runs obtained a gap no greater than 15%. By considering this term, the runs' gap reduces dramatically. This term of the forth criterion is again of high importance on the optimization process.

Special remark for run 15 that considers all terms of all four criteria and obtained an overall gap of 0.48%, which is still good. Accordingly, this run obtained a solution that has no term with better value than the corresponding solution of the initial model. There are other runs which have no term with better value than the corresponding solution of the initial model (see runs 20, 22, 26, and 27). Particularly, run 20 obtain a worst value for all terms of the objective function (on the normalized model in comparison with the initial model). Nonetheless, the normalized model obtained a better solution (regarding the terms' value) for run 3 and 13 than the initial model. The remaining runs obtained better results for some terms with the initial model, and better results for the other terms with the normalized model, leaving no room to arrive to some conclusion about the normalized model having a better performance than the first. Regardless, on average, the normalized model obtained better results for the first and third criteria, and the last term of the fourth criterion, and the initial model obtained better results for the remaining terms (second criterion, and all the terms of the forth criterion excluding the last one).

5.2 Second and third instance results

Similarly, to what was presented regarding the first instance, the results of the second and third instance are exhibit bellow. Figure 5.3 and Figure 5.4 show, similar to Figure 5.1, the results obtained for the second and third instance, respectively, before normalizing the objective function.

On the 35 algorithm runs with the second instance, the model is able to find 17 times (49%) an optimal solution in less than one hour of computation (232s on average). On an overall

	Relative importance of each term of the objective function																				
1st OC: Wmean	1	0	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8	1/8
1st OC: Wad	0	1	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8	1/8
2nd OC: Wroom	0	0	1	0	0	0	0	0	0	0	0	0	0	1/4	1/5	1/2	1/3	1/4	1/4	1/4	0
3rd OC: Wdev	0	0	0	1	0	0	0	0	0	0	0	0	1/3	1/4	1/5	0	1/3	1/4	1/4	0	1/4
4th OC: Wdev^SRG_less	0	0	0	0	1	0	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8	1/8
4th OC: Wdev^SRG_more	0	0	0	0	0	1	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8	1/8
4th OC: Wdev^SPC_less	0	0	0	0	0	0	1	0	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8	1/8
4th OC: Wdev^SPC_more	0	0	0	0	0	0	0	1	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8	1/8
	Best solution found																				
Best bound	39,0474	9,4896	6	0	0	14125	0	0	50,0320	14125	0	15685	15685	15697	15759,4145	57,3585	57,3524	14198,6906	68,3322	15757,8629	15750,3806
Objective value	39,1871	9,7930	6	0	0	14125	0	0	50,7560	14125	0	15685	15685	15697	15772,5559	64,1278	65,6445	14217,3754	83,8918	15775,9610	15750,3806
Gap	0,36%	3,10%	0%	0%	0%	0%	0%	0%	1,43%	0%	0%	0%	0%	0%	0,08%	10,56%	12,63%	0,13%	18,55%	0,11%	0%
Solution time	3600s	3600s	255.25s	0.40s	2.57s	6.80s	1.29s	1.72s	3600s	3.08s	6.20s	9.52s	8.13s	2720.66s	3600s	3600s	3600s	3600s	3600s	3600s	683.54s
Run number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	Obtained value for each term of the objective function																				
1st OC: mean	39,1871	53	61	98	101	92	97	92	39,8727	82	80	79	78	83	49,9568	41,9360	41,9744	52,5340	48,2443	52,0541	49,5771
1st OC: ad	45	9,7930	46	69	75	72	73	66	10,8833	64	72	60	64	61	15,5991	12,1917	11,6701	16,8413	14,6475	15,9069	15,8034
2nd OC: room	91	91	6	91	91	91	91	91	91	91	91	91	91	12	22	10	12	23	21	23	91
3rd OC: dev_more	3	2	3	0	1	1	0	1	0	3	0	2	0	0	0	2	0	0	0	1	0
4th OC: dev^SRG_less	6790	6763	4957	4538	0	0	2570	3541	6725	0	1693	0	0	0	0	5999	5495	0	2446	0	0
4th OC: dev^SRG_more	21095	21608	19382	26523	16765	14125	19155	19466	22050	14125	19478	14305	15685	14845	15205	20904	20700	14125	18131	15265	15685
4th OC: dev^SPC_less	6616	6742	4532	4378	0	0	0	2599	6481	0	0	0	0	0	0	5845	5330	0	0	0	0
4th OC: dev^SPC_more	1220,5	900,5	1062,5	0,5	840	1740	2056,5	0	420,5	1740	0	1380	0	840	480	747,5	480,5	1740	0	420	0
	Deviation between each term value and the corrresponding best value found																				
1st OC: mean	0%	26,06%	35,76%	60,01%	61,20%	57,41%	59,60%	57,41%	1,72%	52,21%	51,02%	50,40%	49,76%	52,79%	21,56%	6,56%	6,64%	25,41%	18,77%	24,72%	20,96%
1st OC: ad	78,24%	0%	78,71%	85,81%	86,94%	86,40%	86,58%	85,16%	10,02%	84,70%	86,40%	83,68%	84,70%	83,95%	37,22%	19,68%	16,08%	41,85%	33,14%	38,44%	38,03%
2nd OC: room	93,41%	93,41%	0%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	50,00%	72,73%	40,00%	50,00%	73,91%	71,43%	73,91%	93,41%
3rd OC: dev_more	100%	100%	100%	0%	100%	100%	0%	100%	0%	100%	0%	100%	0%	0%	0%	100%	0%	0%	0%	100%	0%
4th OC: dev^SRG_less	100%	100%	100%	100%	0%	0%	100%	100%	100%	0%	100%	0%	0%	0%	0%	100%	100%	0%	100%	0%	0%
4th OC: dev^SRG_more	33,04%	34,63%	27,12%	46,74%	15,75%	0%	26,26%	27,44%	35,94%	0%	27,48%	1,26%	9,95%	4,85%	7,10%	32,43%	31,76%	0%	22,09%	7,47%	9,95%
4th OC: dev^SPC_less	100%	100%	100%	100%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0%	0%	100%	100%	0%	0%	0%	0%
4th OC: dev^SPC_more	100%	100%	100%	100%	100%	100%	100%	0%	100%	100%	0%	100%	0%	100%	100%	100%	100%	100%	0%	100%	0%

	Relative importance of each term of the objective function																		
1st OC: Wmean	1/5	0	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10					
1st OC: Wad	0	1/5	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10					
2nd OC: Wroom	1/5	1/5	1/5	1/5	1/5	1/5	0	0	0	0	1/4	1/4	1/5	1/5					
3rd OC: Wdev	1/5	1/5	1/5	1/5	1/5	1/5	0	0	1/3	1/3	1/4	1/4	1/5	1/5					
4th OC: Wdev^SRG_less	1/10	1/10	0	1/5	1/10	1/10	1/2	0	1/3	0	1/4	0	1/5	0					
4th OC: Wdev^SRG_more	1/10	1/10	1/5	0	1/10	1/10	0	1/2	0	1/3	0	1/4	0	1/5					
4th OC: Wdev^SPC_less	1/10	1/10	1/10	1/10	0	1/5	1/2	0	1/3	0	1/4	0	1/5	0					
4th OC: Wdev^SPC_more	1/10	1/10	1/10	1/10	1/5	0	0	1/2	0	1/3	0	1/4	0	1/5					
	Best solution found														MIN	MAX	MEAN	MEDIAN	Number of optimal solutions
Best bound	15742,5319	15706,1716	15755,0153	76,5154	15759,9467	14198,3210	0	15685	0	15685	12	15691,3722	75,8349	15754,4385	0	15759,9467	7920,3745	14125	
Objective value	15751,0844	15725,3547	15775,4965	84,7923	15774,5500	14214,8617	0	15685	0	15685	12	15697	83,7225	15771,7733	0	15776	7926,0945	14125	
Gap	0,05%	0,12%	0,13%	9,76%	0,09%	0,12%	0%	0%	0%	0%	0%	0,04%	9,42%	0,11%	0%	18,55%	1,91%	0,04%	17
Solution time	3600s	3600s	3600s	3600s	3600s	3600s	2.74s	4.07s	2.46s	3.63s	235.5s	3600s	3600s	3600s					
Run number	22	23	24	25	26	27	28	29	30	31	32	33	34	35					
	Obtained value for each term of the objective function														MIN	MAX	MEAN	MEDIAN	DIFFERENCE
1st OC: mean	51,0844	65	51,0801	50,8871	51,4806	52,0082	83	78	97	79	81	87	51,8909	52,1994	39,1871	101	66,9134	61	61,8129
1st OC: ad	57	18,3547	15,4165	15,9052	16,0694	16,8535	74	69	76	66	62	64	16,8316	15,5740	9,7930	76	42,0955	46	66,2070
2nd OC: room	15	22	24	18	22	21	91	91	91	91	12	12	15	19	6	91	53,0286	24	85
3rd OC: dev_more	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	3	0,6571	0	3
4th OC: dev^SRG_less	0	0	1303	0	0	0	0	734,5	0	833	0	867,5	0	1197	0	6790	1612,914286	0	6790
4th OC: dev^SRG_more	15265	15685	15608	15745	15685	14125	16705	15159,5	17905	15558	14785	15112,5	15745	15562	14125	26523	17064,7714	15685	12398
4th OC: dev^SPC_less	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6742	1214,9429	0	6742
4th OC: dev^SPC_more	420	0	77	0	0	1740	1740	525,5	1380	127	1320	572,5	0	123	0	2056,5	688,4	480,5	2056,5
	Deviation between each term value and the corrresponding best value found														MIN	MAX	MEAN	MEDIAN	
1st OC: mean	23,29%	39,71%	23,28%	22,99%	23,88%	24,65%	52,79%	49,76%	59,60%	50,40%	51,62%	54,96%	24,48%	24,93%	0%	61,20%	36,18%	35,76%	
1st OC: ad	82,82%	46,65%	36,48%	38,43%	39,06%	41,89%	86,77%	85,81%	87,11%	85,16%	84,20%	84,70%	41,82%	37,12%	0%	87,11%	60,68%	78,71%	
2nd OC: room	60,00%	72,73%	75,00%	66,67%	72,73%	71,43%	93,41%	93,41%	93,41%	93,41%	50,00%	50,00%	60,00%	68,42%	0%	93,41%	76,20%	75,00%	
3rd OC: dev_more	0%	0%	0%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0,00%	0%	0%	100%	34,29%	0%	
4th OC: dev^SRG_less	0%	0%	100%	0%	0%	0%	0%	100%	0%	100%	0%	100%	0,00%	100%	0%	100%	45,71%	0%	
4th OC: dev^SRG_more	7,47%	9,95%	9,50%	10,29%	9,95%	0%	15,44%	6,82%	21,11%	9,21%	4,46%	6,53%	10,29%	9,23%	0%	46,74%	15,19%	9,95%	
4th OC: dev^SPC_less	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0,00%	0%	0%	100%	22,86%	0%	
4th OC: dev^SPC_more	100%	0%	100%	0%	0%	100%	100%	100%	100%	100%	100%	100%	0,00%	100%	0%	100%	74,29%	100%	

Figure 5.3: Solutions obtained for the second instance, after 35 algorithm runs

	Relative importance of each term of the objective function																			
1st OC: Wmean	1	0	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8
1st OC: Wad	0	1	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8
2nd OC: Wroom	0	0	1	0	0	0	0	0	0	0	0	0	0	1/4	1/5	1/2	1/3	1/4	1/4	1/4
3rd OC: Wdev	0	0	0	1	0	0	0	0	0	0	0	0	1/3	1/4	1/5	0	1/3	1/4	1/4	0
4th OC: Wdev^SRG_less	0	0	0	0	1	0	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8
4th OC: Wdev^SRG_more	0	0	0	0	0	1	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8
4th OC: Wdev^SPC_less	0	0	0	0	0	0	1	0	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8
4th OC: Wdev^SPC_more	0	0	0	0	0	0	0	1	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8
	Best solution found																			
Best bound	41,2108	10,6545	6	0	0	12372,5	0	0	53	12372,5	0	13932,5	13932,5	13941,7624	14010,2453	59,9104	59,8975	12451,5061	71,5815	14010,5160
Objective value	41,421426	10,8930	6	0	0	12372,5	0	0	53,5880	12372,5	0	13932,5	13932,5	13943,5	14024,3432	72,1001	71,1289	12469,7233	87,8816	14026,1846
Gap	0,51%	2,19%	0%	0%	0%	0%	0%	0%	1,04%	0%	0%	0%	0%	0,01%	0,10%	16,91%	15,79%	0,15%	18,55%	0,11%
Solution time	3600s	3600s	310.56s	2.23s	3.09s	8.19s	2.53s	1.89s	3600s	7.74s	6.95s	8.51s	7.58s	3600s	3600s	3600s	3600s	3600s	3600s	3600s
Run number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	Obtained value for each term of the objective function																			
1st OC: mean	41,4214	51	66	76	99	83	94	91	41,8963	102	93	95	90	80	54,0427	45,4850	44,1486	57,1540	50,6309	55,8947
1st OC: ad	48	10,8930	56	66	68	69	70	66	11,6917	76	76	71	67	67	17,8005	13,6151	12,9803	19,0693	15,2507	17,7899
2nd OC: room	91	91	6	91	91	91	91	91	91	91	91	91	91	11	20	13	14	20	22	20
3rd OC: dev_more	3	2	4	0	0	3	5	1	1	4	0	0	0	0	0	3	0	1	0	0
4th OC: dev^SRG_less	7437	8110	4792	5348	0	0	4085	4209	7655	0	3009	0	0	0	0	6408	6342	0	3454	0
4th OC: dev^SRG_more	20049,5	21142,5	17164,5	18080,5	15312,5	12372,5	16637,5	17601,5	21107,5	12372,5	19341,5	13512,5	13932,5	13092,5	13932,5	19020,5	19314,5	12372,5	17386,5	13932,5
4th OC: dev^SPC_less	7241	7841	3876	3199	0	0	0	2854,5	7655	0	0	0	0	0	0	5420	5910	0	0	0
4th OC: dev^SPC_more	1127,5	902,5	983,5	795,5	840	1680	1357	0	482,5	2760	0	420	0	840	0	968,5	630,5	1740	0	0
	Deviation between each term value and the corrresponding best value found																			
1st OC: mean	0%	18,78%	37,24%	45,50%	58,16%	50,09%	55,93%	54,48%	1,13%	59,39%	55,46%	56,40%	53,98%	48,22%	23,35%	8,93%	6,18%	27,53%	18,19%	25,89%
1st OC: ad	77,31%	0%	80,55%	83,50%	83,98%	84,21%	84,44%	83,50%	6,83%	85,67%	85,67%	84,66%	83,74%	83,74%	38,80%	19,99%	16,08%	42,88%	28,57%	38,77%
2nd OC: room	93,41%	93,41%	0%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	45,45%	70,00%	53,85%	57,14%	70,00%	72,73%	70,00%
3rd OC: dev_more	100%	100%	100%	0%	0%	100%	100%	100%	100%	100%	0%	0%	0%	0%	0%	100%	0%	100%	0%	0%
4th OC: dev^SRG_less	100%	100%	100%	100%	0%	0%	100%	100%	100%	0%	100%	0%	0%	0%	0%	100%	100%	0%	100%	0%
4th OC: dev^SRG_more	38,29%	41,48%	27,92%	31,57%	19,20%	0%	25,63%	29,71%	41,38%	0%	36,03%	8,44%	11,20%	5,50%	11,20%	34,95%	35,94%	0%	28,84%	11,20%
4th OC: dev^SPC_less	100%	100%	100%	100%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0%	0%	100%	100%	0%	0%	0%
4th OC: dev^SPC_more	100%	100%	100%	100%	100%	100%	100%	0%	100%	100%	0%	100%	0%	100%	0%	100%	100%	100%	0%	0%

	Relative importance of each term of the objective function																		
1st OC: Wmean	1/8	1/5	0	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10				
1st OC: Wad	1/8	0	1/5	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10				
2nd OC: Wroom	0	1/5	1/5	1/5	1/5	1/5	1/5	0	0	0	0	1/4	1/4	1/5	1/5				
3rd OC: Wdev	1/4	1/5	1/5	1/5	1/5	1/5	1/5	0	0	1/3	1/3	1/4	1/4	1/5	1/5				
4th OC: Wdev^SRG_less	1/8	1/10	1/10	0	1/5	1/10	1/10	1/2	0	1/3	0	1/4	0	1/5	0				
4th OC: Wdev^SRG_more	1/8	1/10	1/10	1/5	0	1/10	1/10	0	1/2	0	1/3	0	1/4	0	1/5				
4th OC: Wdev^SPC_less	1/8	1/10	1/10	1/10	1/10	0	1/5	1/2	0	1/3	0	1/4	0	1/5	0				Number of optimal solutions
4th OC: Wdev^SPC_more	1/8	1/10	1/10	1/10	1/10	1/5	0	0	1/2	0	1/3	0	1/4	0	1/5				
	Best solution found															MIN	MAX	MEAN	MEDIAN	
Best bound	14001,2785	13994,9527	13953,5732	14007,3808	79,6804	14010,2804	12450,2121	0	13932,5	0	13932,5	11	13939,0421	80,4737	14008,1011	0	14010,516	7020,7798	12373	
Objective value	14002,7504	14000,2563	13976,5162	14026,3889	92,3377	14026,6210	12705,4125	0	13932,5	0	13932,5	11	13943,5	89,3742	14028,8537	0	14029	7033,8507	12373	
Gap	0,01%	0,04%	0,16%	0,14%	13,71%	0,12%	2,01%	0%	0%	0%	0%	0%	0,03%	9,96%	0,15%	0%	18,55%	2,33%	0,03%	15
Solution time	3600s	3600s	3600s	3600s	3600s	3600s	3600s	2.13s	8.82s	2.45s	9.20s	185.6s	3600s	3600s	3600s					
Run number	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35					
	Obtained value for each term of the objective function															MIN	MAX	MEAN	MEDIAN	DIFFERENCE
1st OC: mean	53,0245	54,7563	68	54,1186	56,1207	53,8109	55,5203	104	89	93	86	85	80	54,5215	54,1464	41,4214	104	70,0484	66	62,5786
1st OC: ad	17,2259	65	20,0162	17,7703	18,2170	18,3101	19,3922	71	69	73	80	77	65	17,8528	18,2074	10,8930	80	44,7452	56	69,1070
2nd OC: room	91	13	24	22	18	22	28	91	91	91	91	11	11	17	24	6	91	53,2286	28	85
3rd OC: dev_more	0	0	0	0	0	0	0	2	3	0	0	0	0	0	0	0	5	0,9143	0	5
4th OC: dev^SRG_less	0	0	0	772,5	0	0	25	0	988	0	454	0	433,5	0	1170,5	0	8110	1848,3571	0	8110
4th OC: dev^SRG_more	13932,5	13512,5	13932,5	13865	13992,5	13512,5	12577,5	15912,5	13600,5	16032,5	13906,5	13692,5	12986	14052,5	13783	12372,5	21142,5	15342	13932,5	8770
4th OC: dev^SPC_less	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7841	1257,0429	0	7841
4th OC: dev^SPC_more	0	420	0	67,5	0	420	1380	2640	332	420	26	420	946,5	0	149,5	0	2760	649,9714	420	2760
	Deviation between each term value and the corrresponding best value found															MIN	MAX	MEAN	MEDIAN	
1st OC: mean	21,88%	24,35%	39,09%	23,46%	26,19%	23,02%	25,39%	60,17%	53,46%	55,46%	51,84%	51,27%	48,22%	24,03%	23,50%	0%	60,17%	35,89%	37,24%	
1st OC: ad	36,76%	83,24%	45,58%	38,70%	40,20%	40,51%	43,83%	84,66%	84,21%	85,08%	86,38%	85,85%	83,24%	38,98%	40,17%	0%	86,38%	60,29%	80,55%	
2nd OC: room	93,41%	53,85%	75,00%	72,73%	66,67%	72,73%	78,57%	93,41%	93,41%	93,41%	93,41%	45,45%	45,45%	64,71%	75,00%	0%	93,41%	76,49%	78,57%	
3rd OC: dev_more	0%	0%	0%	0%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0%	0%	0%	100%	34,29%	0%	
4th OC: dev^SRG_less	0%	0%	0%	100%	0%	0%	100%	0%	100%	0%	100%	0%	100%	0%	100%	0%	100%	48,57%	0%	
4th OC: dev^SRG_more	11,20%	8,44%	11,20%	10,76%	11,58%	8,44%	1,63%	22,25%	9,03%	22,83%	11,03%	9,64%	4,72%	11,96%	10,23%	0%	41,48%	17,24%	11,20%	
4th OC: dev^SPC_less	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%	22,86%	0%	
4th OC: dev^SPC_more	0%	100%	0%	100%	0%	100%	100%	100%	100%	100%	100%	100%	100%	0%	100%	0%	100%	71,43%	100%	

Figure 5.4: Solutions obtained for the third instance, after 35 algorithm runs

perspective, the model obtained the worst gap of 18.55% (see run 19), an average gap of 1.91% and a median gap of 0.04%, within a computing time limit of one hour. In comparison with the first instance, this instance found one more optimal solution (and thus the average gap of optimal solutions increased). This instance obtained a lower worst gap (-3.77%), a lower average gap (-0.51%), and an equal median gap. Note that the worst gap is obtained for the same run, and that this increment of performance was expected since this instance is oldest and so, smaller than the one analyzed before.

On the 35 algorithm runs with the third instance, the model is able to find 15 times (43%) an optimal solution in less than one hour of computation (38s on average). On an overall perspective, the model obtained the worst gap of 18.55% (see run 19), an average gap of 2.33% and a median gap of 0.03%, within a computing time limit of one hour. In comparison with the first and second instance, this instance found less one and two optimal solutions, respectively (and thus the average gap of optimal solutions decreased). This instance obtained the same worst gap as the second instance (-3.77% than the first instance), a lower average gap in comparison with the first instance (-0.09%), a higher average gap in comparison with the second instance (+0.42%), and a lower median gap in comparison with both instances (-0.01%). Note that the worst gap over all instances is obtained for the same run.

By analyzing the results obtained from the various algorithm runs, for the second and third instances, one can again conclude the same as when analyzing the first instance. The model can easily find the solution when the terms of the second, third and fourth criteria are considered individually or together (see runs 3-8, 10-14, and 28-33). However, when the first criterion is considered, separated or together with at least one of the remaining criteria, the model shows difficulties in finding an optimal solution (see runs 1-2, 9, 15-20, 22-27, and 33-35). An exception is run 21 that found an optimal solution for the second instance, and almost found it for the third instance (gap of 0.01%), even considering all the criteria excluding the second one. The worst gaps (greater than 5%) occur five times (see runs 16, 17, 19, 25, and 34), when optimizing at least the first two criteria without giving weight to $\sum_{s \in S} dev_{s+}^{SRG}$, what makes it again possible to conclude that this term of the forth criterion is of high importance on the optimization process.

Special remark for run 15 that considers all terms of all four criteria and obtained a gap of 0.08% for the second instance, and a gap of 0.10% for the third instance, which is quite good. This run obtained a solution that has a good deviation for the third and the forth criterion, but neglect the first and second criteria. Again, this may be happening because the four optimization criteria of the objective function do not vary in the same order of magnitude. Thus the analysis proceeds with Figure 5.5 and Figure 5.6 that shows, similar to Figure 5.2, the results obtained for the second and third instance, respectively, after normalizing the objective function by the ratio of differences. The best and worst performance of each criterion term were again estimated by the best and worst value obtained over all the 35 algorithm runs for each instance, respectively.

After performing the normalization of the objective function, the model is able to find 16 times (46%) an optimal solution for the second instance, and 15 times for the third instance (43%), in less than one hour of computation (457s and 265s on average, respectively). Unlike the initial model, this normalized model cannot find an optimal solution for run 21, and it cannot conclude that the solution found for run 14 and run 32 is an optimal one. On an overall perspective, the normalized model obtained, for the second and third instance, the worst gap of 67.03% and 78.99% (see run 34 and 19), an average gap of 6.35% and 7.94%, and a median gap of 0.09% and 0.12%, respectively, within a computing time limit of one hour. Again, the difficulty and so the average time needed to find an optimal solution increased significantly as it was expected.

By analyzing the results obtained from the various algorithm runs, for the second and third instance, one can again conclude that the normalized model as the initial model can easily find the solution when the terms of the second, third and fourth criteria are considered individually or together (see runs 3-8, 10-14, and 28-32). An exception is run 33, which the model shows difficulties in finding an optimal solution (obtained a gap of 0.04% for both instances). When the first criterion is considered, separated or together with at least one of the remaining criteria, the normalized model, as the initial model, shows difficulties in finding an optimal solution (see runs 1-2, 9, 15-27, and 34-35). For both instances, the worst gaps (greater than 30%) occur three times, when optimizing at least the first three criteria and one term of the forth criterion, without giving weight to $\sum_{s \in S} dev_{s+}^{SRG}$ (see runs 19, 25, and 34), so this term of the forth criterion is again of high importance on the optimization process.

Special remark for run 15 that considers all terms of all four criteria and obtained an overall gap of 0.24% and 3.44% for the second and third instance, respectively, which is quite good for the second instance, but disappointing for the third instance. This run obtained a solution that only has two terms (the third and the eight term) with better value than the corresponding solution of the initial model for the second instance, and none for the third instance. There are other runs which have no term with better value than the corresponding solution of the initial model (see runs 5, 6, 13, 25-27, and 34 for the second instance, and runs 13, 15, 25 and 27 for the third instance). Nonetheless, regarding the second instance, the normalized model obtained a better solution (regarding the terms' value) for runs 23, 28 and 30 than the initial model, while the third instance obtained a better solution only for run 4. The remaining runs obtained better results for some terms with the initial model, and better results for the other terms with the normalized model, leaving no room to arrive to some conclusion about the normalized model having a better performance than the first. Regardless, on average, the normalized model with the second instance obtained better results for the eighth term, and the initial model obtained

	Relative importance of each term of the objective function										
1st OC: Wmean	1	0	0	0	0	0	0	0	1/2	0	0
1st OC: Wad	0	1	0	0	0	0	0	0	1/2	0	0
2nd OC: Wroom	0	0	1	0	0	0	0	0	0	0	0
3rd OC: Wdev	0	0	0	1	0	0	0	0	0	0	0
4th OC: Wdev^SRG_less	0	0	0	0	1	0	0	0	0	1/2	0
4th OC: Wdev^SRG_more	0	0	0	0	0	1	0	0	0	1/2	0
4th OC: Wdev^SPC_less	0	0	0	0	0	0	1	0	0	0	1/2
4th OC: Wdev^SPC_more	0	0	0	0	0	0	0	1	0	0	1/2
	Best solution found										
Best bound	39,0474	9,4896	6	0	0	14125	0	0	50,0320	14125	0
Objective value	39,2084	9,9161	6	0	0	14125	0	0	50,8857	14125	0
Gap	0,41%	4,30%	0%	0%	0%	0%	0%	0%	1,68%	0%	0%
Solution time	3600s	3600s	54.41s	2.97s	2.39s	8.36s	3.42s	0.35s	3600s	3.31s	7.89s
Run number	1	2	3	4	5	6	7	8	9	10	11
	Obtained value for each term of the objective function										
1st OC: mean	39,2084	53	75	80	101	98	95	96	39,9895	91	83
1st OC: ad	45	9,9161	57	58	78	79	67	73	10,8963	63	69
2nd OC: room	91	91	6	91	91	91	91	91	91	91	91
3rd OC: dev_more	2	1	3	0	2	3	6	0	3	1	0
4th OC: dev^SRG_less	6730	6537	4799	5184	0	0	3111	4026	6924	0	1317
4th OC: dev^SRG_more	21575	21862	19104	19309	17965	14125	17536	26011	21349	14125	18082
4th OC: dev^SPC_less	6490	6536	4319	3152	0	0	0	4000	6363	0	0
4th OC: dev^SPC_more	680,5	480,5	1214,5	481,5	2220	1740	3156	0	947,5	1800	0
	Deviation between each term value and the corrresponding best value found										
1st OC: mean	0,05%	26,06%	47,75%	51,02%	61,20%	60,01%	58,75%	59,18%	2,01%	56,94%	52,79%
1st OC: ad	78,24%	1,24%	82,82%	83,12%	87,44%	87,60%	85,38%	86,58%	10,13%	84,46%	85,81%
2nd OC: room	93,41%	93,41%	0%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%
3rd OC: dev_more	100%	100%	100%	0%	100%	100%	100%	0%	100%	100%	0%
4th OC: dev^SRG_less	100%	100%	100%	100%	0%	0%	100%	100%	100%	0%	100%
4th OC: dev^SRG_more	34,53%	35,39%	26,06%	26,85%	21,37%	0%	19,45%	45,70%	33,84%	0%	21,88%
4th OC: dev^SPC_less	100%	100%	100%	100%	0%	0%	0%	100%	100%	0%	0%
4th OC: dev^SPC_more	100%	100%	100%	100%	100%	100%	100%	0%	100%	100%	0%

	Relative importance of each term of the objective function									
1st OC: Wmean	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8	1/8
1st OC: Wad	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8	1/8
2nd OC: Wroom	0	0	1/4	1/5	1/2	1/3	1/4	1/4	1/4	0
3rd OC: Wdev	0	1/3	1/4	1/5	0	1/3	1/4	1/4	0	1/4
4th OC: Wdev^SRG_less	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8	1/8
4th OC: Wdev^SRG_more	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8	1/8
4th OC: Wdev^SPC_less	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8	1/8
4th OC: Wdev^SPC_more	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8	1/8
	Best solution found									
Best bound	15685	15685	15697	15759,4145	57,3585	57,3524	14198,6906	68,3322	15757,8629	15750,3806
Objective value	15685	15685	15697	15798,1175	69,1276	68,2871	16682,4325	112,4126	16906,1850	15793,2047
Gap	0%	0%	0%	0,24%	17,03%	16,01%	14,89%	39,21%	6,79%	0,27%
Solution time	3.62s	4.08s	3600s	3600s	3600s	3600s	3600s	3600s	3600s	3600s
Run number	12	13	14	15	16	17	18	19	20	21
	Obtained value for each term of the objective function									
1st OC: mean	95	88	77	51,0417	40,7621	42,3416	61,9461	48,6047	52,3677	48,8288
1st OC: ad	71	66	60	16,0758	11,3655	11,9455	20,4864	14,8079	16,3173	15,3759
2nd OC: room	91	91	12	20	17	14	21	20	24	91
3rd OC: dev_more	0	0	0	0	2	0	0	0	2	0
4th OC: dev^SRG_less	0	0	0	13	6572	5353	867	2570	904,5	22
4th OC: dev^SRG_more	15685	15685	15685	15698	21417	20678	15712	18255	15869,5	15707
4th OC: dev^SPC_less	0	0	0	0	6099	4613	313	29	17,5	0
4th OC: dev^SPC_more	0	0	0	0	680,5	480	1170	0	22	0
	Deviation between each term value and the corrresponding best value found									
1st OC: mean	58,75%	55,47%	49,11%	23,23%	3,86%	7,45%	36,74%	19,38%	25,17%	19,75%
1st OC: ad	86,21%	85,16%	83,68%	39,08%	13,84%	18,02%	52,20%	33,87%	39,98%	36,31%
2nd OC: room	93,41%	93,41%	50,00%	70,00%	64,71%	57,14%	71,43%	70,00%	75,00%	93,41%
3rd OC: dev_more	0%	0%	0%	0%	100%	0%	0%	0%	100%	0%
4th OC: dev^SRG_less	0%	0%	0%	100%	100%	100%	100%	100%	100%	100%
4th OC: dev^SRG_more	9,95%	9,95%	9,95%	10,02%	34,05%	31,69%	10,10%	22,62%	10,99%	10,07%
4th OC: dev^SPC_less	0%	0%	0%	0%	100%	100%	100%	100%	100%	0%
4th OC: dev^SPC_more	0%	0%	0%	0%	100%	100%	100%	0%	100%	0%

	Relative importance of each term of the objective function																		
1st OC: Wmean	1/5	0	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10					
1st OC: Wad	0	1/5	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10					
2nd OC: Wroom	1/5	1/5	1/5	1/5	1/5	1/5	0	0	0	0	1/4	1/4	1/5	1/5					
3rd OC: Wdev	1/5	1/5	1/5	1/5	1/5	1/5	0	0	1/3	1/3	1/4	1/4	1/5	1/5					
4th OC: Wdev^SRG_less	1/10	1/10	0	1/5	1/10	1/10	1/2	0	1/3	0	1/4	0	1/5	0					
4th OC: Wdev^SRG_more	1/10	1/10	1/5	0	1/10	1/10	0	1/2	0	1/3	0	1/4	0	1/5					
4th OC: Wdev^SPC_less	1/10	1/10	1/10	1/10	0	1/5	1/2	0	1/3	0	1/4	0	1/5	0					
4th OC: Wdev^SPC_more	1/10	1/10	1/10	1/10	1/5	0	0	1/2	0	1/3	0	1/4	0	1/5					
	Best solution found														MIN	MAX	MEAN	MEDIAN	Number of optimal solutions
Best bound	15742,5319	15706,1716	15755,0153	76,5154	15759,9467	14198,3210	0	15685	0	15685	12	15691,3722	75,8349	15754,4385	0	15759,9467	7920,3745	14125	
Objective value	15763,1897	15720,7707	16672,3695	121,5347	16074,0653	14643,1586	0	15685	0	15685	12	15697	229,9864	16852,4228	0	16906	8114,5507	14125	
Gap	0,13%	0,09%	5,50%	37,04%	1,95%	3,04%	0%	0%	0%	0%	0%	0,04%	67,03%	6,52%	0%	67,03%	6,35%	0,09%	16
Solution time	3600s	3600s	3600s	3600s	3600s	3600s	3.01s	10.10s	3.02s	5.38s	3600s	3600s	3600s	3600s					
Run number	22	23	24	25	26	27	28	29	30	31	32	33	34	35					
	Obtained value for each term of the objective function														MIN	MAX	MEAN	MEDIAN	DIFFERENCE
1st OC: mean	50,1897	59	51,9002	52,3445	53,0647	55,7613	81	86	85	89	83	79	52,7869	51,8567	39,2084	101	68,1998	61,9461	61,7916
1st OC: ad	54	15,7707	15,4693	16,1902	17,0006	17,8973	68	69	68	72	58	70	17,1994	16,0661	9,9161	79	42,5080	54	69,0839
2nd OC: room	20	20	22	21	34	26	91	91	91	91	12	12	29	37	6	91	54,6857	37	85
3rd OC: dev_more	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	6	0,7429	0	6
4th OC: dev^SRG_less	4	0	1653	32	52,5	160	0	0	0	0	0	950	131	2082,5	0	6924	1714,128571	131	6924
4th OC: dev^SRG_more	15689	15685	16378	16017	15917,5	14285	14845	15685	15325	15685	14425	15675	16056	16747,5	14125	26011	17138,5571	15869,5	11886
4th OC: dev^SPC_less	0	0	179	0	17,5	98,5	0	0	0	0	0	0	0	475,5	0	6536	1220,0571	0	6536
4th OC: dev^SPC_more	0	0	26	0	0	1860	840	0	480	0	1380	10	0	0	0	3156	561,9714	22	3156
	Deviation between each term value and the corrresponding best value found														MIN	MAX	MEAN	MEDIAN	
1st OC: mean	21,92%	33,58%	24,50%	25,14%	26,15%	29,72%	51,62%	54,43%	53,90%	55,97%	52,79%	50,40%	25,76%	24,43%	0%	61,20%	37,28%	36,74%	
1st OC: ad	81,86%	37,90%	36,69%	39,51%	42,40%	45,28%	85,60%	85,81%	85,60%	86,40%	83,12%	86,01%	43,06%	39,05%	1,24%	87,60%	61,13%	81,86%	
2nd OC: room	70,00%	70,00%	72,73%	71,43%	82,35%	76,92%	93,41%	93,41%	93,41%	93,41%	50,00%	50,00%	79,31%	83,78%	0%	93,41%	78,65%	83,78%	
3rd OC: dev_more	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	100%	31,43%	0%	
4th OC: dev^SRG_less	100%	0%	100%	100%	100%	100%	0%	0%	0%	0%	0%	100%	100%	100%	0%	100%	65,71%	100%	
4th OC: dev^SRG_more	9,97%	9,95%	13,76%	11,81%	11,26%	1,12%	4,85%	9,95%	7,83%	9,95%	2,08%	9,89%	12,03%	15,66%	0%	45,70%	15,84%	10,99%	
4th OC: dev^SPC_less	0%	0%	100%	0%	100%	100%	0%	0%	0%	0%	0%	0%	0%	100%	0%	100%	42,86%	0%	
4th OC: dev^SPC_more	0%	0%	100%	0%	0%	100%	100%	0%	100%	0%	100%	100%	0%	0%	0%	100%	54,29%	100%	

Figure 5.5: Solutions obtained for the second instance, after 35 algorithm runs of the normalized objective function

	Relative importance of each term of the objective function																				
1st OC: Wmean	1	0	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8	1/8
1st OC: Wad	0	1	0	0	0	0	0	0	1/2	0	0	0	0	0	1/10	1/4	1/6	1/8	1/8	1/8	1/8
2nd OC: Wroom	0	0	1	0	0	0	0	0	0	0	0	0	0	1/4	1/5	1/2	1/3	1/4	1/4	1/4	0
3rd OC: Wdev	0	0	0	1	0	0	0	0	0	0	0	0	1/3	1/4	1/5	0	1/3	1/4	1/4	0	1/4
4th OC: Wdev^SRG_less	0	0	0	0	1	0	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8	1/8
4th OC: Wdev^SRG_more	0	0	0	0	0	1	0	0	0	1/2	0	1/4	1/6	1/8	1/10	0	0	1/8	0	1/8	1/8
4th OC: Wdev^SPC_less	0	0	0	0	0	0	1	0	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8	1/8
4th OC: Wdev^SPC_more	0	0	0	0	0	0	0	1	0	0	1/2	1/4	1/6	1/8	1/10	0	0	0	1/8	1/8	1/8
	Best solution found																				
Best bound	41,2108	10,6545	6	0	0	12372,5	0	0	53,0332	12372,5	0	13932,5	13932,5	13941,7624	14010,2453	59,9104	59,8975	12451,5061	71,5815	14010,5160	14001,2785
Objective value	41,4617	10,7576	6	0	0	12372,5	0	0	54,2819	12372,5	0	13932,5	13932,5	13943,5	14509,2391	68,2023	75,6961	12935,9063	340,7221	14255,7087	14018,3216
Gap	0,61%	0,96%	0%	0%	0%	0%	0%	0%	2,30%	0%	0%	0%	0%	0,01%	3,44%	12,16%	20,87%	3,74%	78,99%	1,72%	0,12%
Solution time	3600s	3600s	293.68s	2.50s	2.49s	7.23s	3.48s	2.19s	3600s	4.84s	5.83s	3.96s	8.64s	3600s	3600s	3600s	3600s	3600s	3600s	3600s	3600s
Run number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	Obtained value for each term of the objective function																				
1st OC: mean	41,4617	51	85	74	97	91	89	100	42,3381	92	89	82	90	97	56,6142	42,8589	43,7541	55,0766	50,6920	55,2747	52,6869
1st OC: ad	46	10,7576	57	62	73	72	75	72	11,9438	69	72	70	71	74	18,6249	12,3434	12,9420	18,3296	16,5301	18,4340	17,1348
2nd OC: room	91	91	6	91	91	91	91	91	91	91	91	91	91	11	33	13	19	33	33	31	91
3rd OC: dev_more	3	2	4	0	2	2	3	0	2	5	1	0	0	0	0	2	0	0	0	0	0
4th OC: dev^SRG_less	7226	8078	4248	4590	0	0	4253	4852	7264	0	1795	0	0	0	165,5	6692	6585	78,5	3744	95,5	8
4th OC: dev^SRG_more	19898,5	21170,5	16920,5	17742,5	12972,5	12372,5	17345,5	18064,5	20356,5	12372,5	17587,5	13932,5	13932,5	13932,5	14098	19724,5	19617,5	12751	16356,5	14028	13940,5
4th OC: dev^SPC_less	6978	7961	3184	2853	0	0	0	3442,5	7264	0	0	0	0	0	137,5	6281	6240	0	240	27,5	0
4th OC: dev^SPC_more	1067,5	842,5	1327	421	1860	2580	2658,5	0	902,5	2580	0	0	0	0	0	902	902,5	1380	0,5	0	0
	Deviation between each term value and the corrresponding best value found																				
1st OC: mean	0,10%	18,78%	51,27%	44,03%	57,30%	54,48%	53,46%	58,58%	2,17%	54,98%	53,46%	49,49%	53,98%	57,30%	26,84%	3,35%	5,33%	24,79%	18,29%	25,06%	21,38%
1st OC: ad	76,32%	-1,26%	80,89%	82,43%	85,08%	84,87%	85,48%	84,87%	8,80%	84,21%	84,87%	84,44%	84,66%	85,28%	41,51%	11,75%	15,83%	40,57%	34,10%	40,91%	36,43%
2nd OC: room	93,41%	93,41%	0%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	93,41%	45,45%	81,82%	53,85%	68,42%	81,82%	81,82%	80,65%	93,41%
3rd OC: dev_more	100%	100%	100%	0%	100%	100%	100%	0%	100%	100%	100%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%
4th OC: dev^SRG_less	100%	100%	100%	100%	0%	0%	100%	100%	100%	0%	100%	0%	0%	0%	100%	100%	100%	100%	100%	100%	100%
4th OC: dev^SRG_more	37,82%	41,56%	26,88%	30,27%	4,63%	0%	28,67%	31,51%	39,22%	0%	29,65%	11,20%	11,20%	11,20%	12,24%	37,27%	36,93%	2,97%	24,36%	11,80%	11,25%
4th OC: dev^SPC_less	100%	100%	100%	100%	0%	0%	0%	100%	100%	0%	0%	0%	0%	0%	100%	100%	100%	0%	100%	100%	0%
4th OC: dev^SPC_more	100%	100%	100%	100%	100%	100%	100%	0%	100%	100%	0%	0%	0%	0%	0%	100%	100%	100%	100%	0%	0%

	Relative importance of each term of the objective function													
1st OC: Wmean	1/5	0	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10
1st OC: Wad	0	1/5	1/10	1/10	1/10	1/10	0	0	0	0	0	0	1/10	1/10
2nd OC: Wroom	1/5	1/5	1/5	1/5	1/5	1/5	0	0	0	0	1/4	1/4	1/5	1/5
3rd OC: Wdev	1/5	1/5	1/5	1/5	1/5	1/5	0	0	1/3	1/3	1/4	1/4	1/5	1/5
4th OC: Wdev^SRG_less	1/10	1/10	0	1/5	1/10	1/10	1/2	0	1/3	0	1/4	0	1/5	0
4th OC: Wdev^SRG_more	1/10	1/10	1/5	0	1/10	1/10	0	1/2	0	1/3	0	1/4	0	1/5
4th OC: Wdev^SPC_less	1/10	1/10	1/10	1/10	0	1/5	1/2	0	1/3	0	1/4	0	1/5	0
4th OC: Wdev^SPC_more	1/10	1/10	1/10	1/10	1/5	0	0	1/2	0	1/3	0	1/4	0	1/5
	Best solution found													
Best bound	13994,9527	13953,5732	14007,3808	79,6804	14010,2804	12450,2121	0	13932,5	0	13932,5	11	13939,0421	80,4737	14008,1011
Objective value	14013,5812	14030,3308	15805,3518	265,0772	14773,9141	13097,0256	0	13932,5	0	13932,5	11	13944,5	177,1562	14932,6213
Gap	0,13%	0,55%	11,38%	69,94%	5,17%	4,94%	0%	0%	0%	0%	0%	0,04%	54,57%	6,19%
Solution time	3600s	3600s	3600s	3600s	3600s	3600s	2.02s	19.25s	2.71s	10.19s	3600s	3600s	3600s	3600s
Run number	22	23	24	25	26	27	28	29	30	31	32	33	34	35
	Obtained value for each term of the objective function													
1st OC: mean	54,0812	67	55,8981	57,1264	57,2488	62,1238	101	80	86	82	89	85	54,8393	54,3321
1st OC: ad	57	17,8308	17,4537	18,4508	19,1653	20,4019	78	76	69	69	70	66	18,3170	17,7892
2nd OC: room	17	20	31	29	35	44	91	91	91	91	11	12	29	40
3rd OC: dev_more	0	0	0	0	0	0	3	0	0	0	0	0	0	0
4th OC: dev^SRG_less	5	0	1517,5	117	335	179	0	0	0	960	0	0	75	2268
4th OC: dev^SRG_more	13937,5	13992,5	14970	15009,5	14327,5	12791,5	14952,5	13932,5	15492,5	13932,5	12372,5	13932,5	13647,5	14820,5
4th OC: dev^SPC_less	0	0	731	43,5	117	0	0	0	0	0	0	0	0	755,5
4th OC: dev^SPC_more	0	0	0	0	0	1743	3060	0	1380	0	1740	0	480	0
	Deviation between each term value and the corrresponding best value found													
1st OC: mean	23,41%	38,18%	25,90%	27,49%	27,65%	33,32%	58,99%	48,22%	51,84%	49,49%	53,46%	51,27%	24,47%	23,76%
1st OC: ad	80,89%	38,91%	37,59%	40,96%	43,16%	46,61%	86,03%	85,67%	84,21%	84,21%	84,44%	83,50%	40,53%	38,77%
2nd OC: room	64,71%	70,00%	80,65%	79,31%	82,86%	86,36%	93,41%	93,41%	93,41%	93,41%	45,45%	50,00%	79,31%	85,00%
3rd OC: dev_more	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%
4th OC: dev^SRG_less	100%	0%	100%	100%	100%	100%	0%	0%	0%	100%	0%	0%	100%	100%
4th OC: dev^SRG_more	11,23%	11,58%	17,35%	17,57%	13,65%	3,28%	17,25%	11,20%	20,14%	11,20%	0%	11,20%	9,34%	16,52%
4th OC: dev^SPC_less	0%	0%	100%	100%	100%	0%	0%	0%	0%	0%	0%	0%	0%	100%
4th OC: dev^SPC_more	0%	0%	0%	0%	0%	100%	100%	0%	100%	0%	100%	0%	100%	0%

	MIN	MAX	MEAN	MEDIAN	Number of optimal solutions
Best bound	0	14010,5160	7020,7798	12372,5	
Objective value	0	15805,3518	7193,8673	12372,5	
Gap	0%	78,99%	7,94%	0,12%	15

	MIN	MAX	MEAN	MEDIAN	DIFFERENCE
1st OC: mean	41,4617	101	70,3831	67	59,5383
1st OC: ad	10,7576	78	44,6985	57	67,2424
2nd OC: room	6	91	56,9714	44	85
3rd OC: dev_more	0	5	0,8286	0	5
4th OC: dev^SRG_less	0	8078	1860,8857	117	8078
4th OC: dev^SRG_more	12372,5	21170,5	15349,3857	14098	8798
4th OC: dev^SPC_less	0	7961	1321,5857	0	7961
4th OC: dev^SPC_more	0	3060	737,9143	0,5	3060

	MIN	MAX	MEAN	MEDIAN
1st OC: mean	0,10%	58,99%	36,34%	38,18%
1st OC: ad	-1,26%	86,03%	60,21%	80,89%
2nd OC: room	0%	93,41%	80,15%	86,36%
3rd OC: dev_more	0%	100%	31,43%	0%
4th OC: dev^SRG_less	0%	100%	65,71%	100%
4th OC: dev^SRG_more	0%	41,56%	17,49%	12,24%
4th OC: dev^SPC_less	0%	100%	42,86%	0%
4th OC: dev^SPC_more	0%	100%	51,43%	100%

Figure 5.6: Solutions obtained for the third instance, after 35 algorithm runs of the normalized objective function

better results for the remaining terms. On average, the normalized model with the third instance obtained better results for the second and forth terms, and the initial model obtained better results for the remaining terms.

The analysis proceeds with a comparison of the results of the model (without the normalization of the objective function) with the MSS implemented. On an overall perspective, the normalization had not lead our study to better results, since the performance of the normalized model was bellow expected, and it did not bring clear improvements.

5.3 Results comparison

Rather than trying to find the overall best MSS for the hospital under study, which is a subjective matter after all, the analysis proceeds with a comparison of the results of the 35 algorithm runs with the first instance, and the last implemented schedule that was collected from the hospital (for the same time period).

The MSS used, in the operating theatre under study, in the first trimester of 2015, is presented on Figure 5.7, and the glossary of surgical specialty's IDs (that are used on the collected MSS) is given on Table 5.2. The MSS used differs from the ones generated by the MILP model in two ways. The model developed requires (as a structural constraint) that each shift could only be assigned to a maximum of three surgeons or a surgical specialty, or not be assigned. On the collected MSS, the surgical specialties do not need to be assigned to the whole shift (e.g. in room 1, on Monday afternoon, the shift is assigned to two surgeons and a surgical specialty). In addition, a portion of the shift can be assigned to more than one surgeon when they perform surgeries together (e.g. see room 1, on Friday afternoon). The latter "rule" is not considered in our model, since it was not requested by the head doctor of the surgical suite. Notwithstanding, the model does not avoid this from happening.

Table 5.3 exhibits the value of each term of the objective function for the MSS used. By comparing the values on this table with the ones presented on Figure 5.1 (which concerns the solutions obtained for the first instance, after 35 algorithm run), it is reasonable to decide which runs produce quality solutions (i.e. improve the current schedule), and which ones do not.

Algorithm runs which do not consider the second criterion obtained the worst value for this term, 91. This value is way too distant from the value obtained for the same term of the hospital "solution", 28. Given that, the surgical specialties are too dispersed through the rooms, which will cause e.g. an increment of turnover time cause by too much material displacement and increment of cleaning time, and so the solutions obtained for the algorithm runs 1, 2, 4-13, 21, 28-31 will be discarded.

Algorithm runs which do not consider the first two terms of the fourth criterion obtained

Day & Shift \ Room		1		2		3		4		5		6		7	
		Surgical specialty's ID	Surgeon's ID	Surgical specialty's ID	Surgeon's ID	Surgical specialty's ID	Surgeon's ID	Surgical specialty's ID	Surgeon's ID	Surgical specialty's ID	Surgeon's ID	Surgical specialty's ID	Surgeon's ID	Surgical specialty's ID	Surgeon's ID
Monday	**M**			APM	SRG 141	PLS	SRG 105	GYN	SRG 44	ORT	SRG 102	ORT ORT	SRG 40 SRG 193	GES GES	SRG 20 SRG 23
	A	VAS VAS OPH	SRG 144 SRG 68 *(SPC 10)*	OTO OTO	SRG 63 SRG 55	URO	SRG 173	URO URO	SRG 5 SRG 33	ORT	SRG 198	ORT NEU	SRH 196 SRG 21	GYN GYN	SRG 58 SRG 94
Tuesday	**M**	OTO OTO	SRG 2 SRG 208	OTO OTO OTO	SRG 69 SRG 145 SRG 215	GYN	*(SPC 8)*	GYN GYN GYN	SRG 59 SRG 39 *(SPC 8)*	ORT	SRG 30	ORT	SRG 101	GES GES	SRG 88 SRG 140
	A	OPH	SRG 120	OTO OTO OTO	SRG 69 SRG 145 SRG 215	GES GES	SRG 86 SRG 116	GYN GYN GYN	SRG 47 SRG 119 SRG 168	ORT	SRG 65	VAS ORT	SRG 28 SRG 130	GES GES	SRG 88 SRG 140
Wednesday	**M**	OPH OPH	SRG 200 *(SPC 10)*	GYN GYN GYN	SRG 59 SRG 137 SRG 135	OTO OTO	SRG 96 SRG 117	URO URO	SRG 74 SRG 132			GYN GYN NEU	SRG 100 67 *(SPC 9)*	GES	SRG 16
	A	GES GES	SRG 176 SRG 1	OTO OTO	SRG 183 SRG 212	PLS	SRG 105	URO	SRG 6	ORT ORT	SRG 85 SRG 65	NEU NEU NEU	SRG 124 SRG 99 *(SPC 9)*	GES GES	SRG 14 SRG 61
Thrusday	**M**	OPH OPH	SRG 104 *(SPC 10)*	OTO OTO	SRG 69 SRG 145			VAS	SRG 49	ORT	SRG 106	ORT ORT	SRG 102 SRG 25	NEU NEU NEU	SRG 127 SRG 115 *(SPC 9)*
	A	OPH	SRG 120	OTO OTO	SRG 69 SRG 139	GES GES GES	SRG 143 SRG 34 SRG 16	VAS VAS	SRG 49 *(SPC 6)*	ORT ORT	SRG 77 SRG 91	ORT ORT	SRG 204 SRG 48/ SRG 112	NEU NEU MAS	SRG 127 *(SPC 9)* SRG 174
Friday	**M**	VAS	*(SPC 6)*			PLS PLS	SRG 84 SRG 123	URO URO	SRG 150 SRG 152	ORT ORT	SRG 15 *(SPC 11)*	ORT ORT	SRG 30 SRG 52	GYN	*(SPC 8)*
	A	GES OTO OPH	SRG 190 SRG 221/ SRG 218 SRG 219	OTO OTO OTO	SRG 83 SRG 72 SRG 76	PES PES PES	SRG 8 SRG 41/ SRG 146 SRG 154	GES GES	SRG 116 SRG 86	ORT ORT	SRG 4/ SRG 201 SRG 15	ORT ORT ORT	SRG 30 SRG 52 SRG 130	CAS CAS	SRG 79 SRG 156

Figure 5.7: MSS used, in the operating theatre under study, in the first trimester of 2015

higher values for these terms, over 4600 for the first term, and over 16000 for the second. These values are way too distant from the value obtained for the same terms considering the hospital “solution”, 1820 for the first term, and 15091 for the second one. Given that, the surgeons assignment has a strong lagged effect, e.g. the under surgeons’ assignment (first term of the fourth criterion) sums up more than 47 hours per week than the MSS used, and hence the solutions obtained for the algorithm runs 3, 16, 17, and 19 will be discarded.

Table 5.2: Glossary of surgical specialty’s IDs

ID	**Surgical specialty**
APM	Anesthesiology and Pain Medice
CAS	Cardiothoracic Surgery
GES	General Surgery
GYN	Gynecology
MAS	Maxillofacial Surgery
NEU	Neurosurgery
OPH	Ophthalmology
ORT	Orthopedics and Traumatology
OTO	Otorhinolaringology
PES	Pediatric Surgery
PLS	Plastic Surgery
URO	Urology
VAS	Vascular Surgery

Table 5.3: Value of each term of the objective function for the MSS used

Term of the objective function	**Value obtained**
1st OC: mean	91.0346
1st OC: ad	32.6859
2nd OC: room	28
3rd OC: dev_more	1
4th OC: dev^SRG_less	1819.5
4th OC: dev^SRG_more	15090.5
4th OC: dev^SPC_less	0
4th OC: dev^SPC_more	188

Algorithm runs which do not consider both terms of the first criterion obtained too high values for these terms, especially for the second one. The MSS used obtained a value of 33 for the absolute deviation (from the mean) of the number of patients sent to each of the hospitalization units, while problems 14, 32 and 33 value obtained above of 59. Given that, the demand for the downstream units under study is not being correctly leveled over the days, and hence the solutions obtained for these algorithm runs will be discarded. Moreover, by comparing the solution obtained for algorithm run 22 with the solution obtained for algorithm run 23, one can conclude that the solution for the latter is better than the solution for run 22, since there is no interest in lowering the first term of the first criterion (the mean number of patients sent to the hospitalization units) if it increases dramatically the second term of the first criterion (the corresponding absolute deviation). By doing so, the purpose of levelling the demand of the hospitalization units is not achieved. Thus algorithm run 22 is discarded.

The ten remaining solutions (for runs 15, 18, 20, 23-27, 34, and 35) are the ones that have the best performance. Some of these solutions were presented on a meeting with the head doctor of the surgical suite. Others were immediately discarded while discussing the trade-offs on the relevant criteria for building a schedule. However, there is one that stands, run 23. The solution obtained for run 23 was the best value possible for the second criterion without compromising the others, especially the first and third term of the forth criterion, which are of high priority too. It is more relevant to reduce the scattered effect of surgical specialties through the rooms, than increasing the levelling of the number of patients sent on each day to each hospitalization unit, since the latter can be overcome on the next decision level, surgery scheduling. As the workload of the hospital surgical suite is increasing, it is also of high priority to reduce the under assignment of surgeons, while over assigning is not considered as so bad. The third criterion is the one of less importance, since it is very difficult to assign surgical specialty with all the not individually assigned surgeons available. Besides the solution for run 23 not performing so well for the first and third criteria, and the second and fourth terms of the fourth criterion, the solution for run 23 still has better performance for all of them in comparison with the MSS used, and so solution for run 23 improves the implemented schedule. Figure 5.8 presents the MSS obtained from the solution of run 23.

The head doctor of the surgical suite was particularly impressed with the study undertaken. He appreciated the solutions at a first glance but a deep understanding requires further analysis. Therefore, he committed to analyze in more detail the solutions presented, and to provide some feedback on the quality of the solutions and on the possibility of implementing the MSS associated to the selected solution.

Day & Shift	Room	1 Surgical specialty's ID	1 Surgeon's ID	2 Surgical specialty's ID	2 Surgeon's ID	3 Surgical specialty's ID	3 Surgeon's ID	4 Surgical specialty's ID	4 Surgeon's ID	5 Surgical specialty's ID	5 Surgeon's ID	6 Surgical specialty's ID	6 Surgeon's ID	7 Surgical specialty's ID	7 Surgeon's ID
Monday	M	GYN OPH GYN	SRG167 SRG118 SRG53	APM NEU	SRG141 SRG109			OTO ORT	SRG212 SRG102			ORT ORT	SRG198 SRG196	PLS VAS VAS	SRG84 SRG66 SRG49
Monday	A	OPH GYN	SRG142 SRG58	NEU	SRG128			CAS OTO OTO	SRG182 SRG63 SRG55	GYN GYN URO	SRG134 SRG75 SRG33	ORT ORT GES	SRG196 SRG130 SRG86	VAS PLS VAS	SRG144 SRG111 SRG68
Tuesday	M	OPH OPH GYN	SRG120 SRG95 SRG47					OTO ORT ORT	SRG145 SRG65 SRG30	GYN GYN GYN	SRG59 SRG53 SRG24	GES GES	SRG155 SRG88	VAS VAS	SRG151 SRG28
Tuesday	A	GYN OPH GYN	SRG220 SRG36 SRG11	APM MAS NEU	SRG224 SRG174 SRG21			ORT ORT ORT	SRG130 SRG52 SRG30	PES URO GYN	SRG154 SRG150 SRG59	GES ORT GES	SRG155 SRG101 SRG1	VAS PLS	SRG199 SRG105
Wednesday	M	GYN GYN	SRG44 SRG17	NEU NEU	SRG217 SRG21			OTO ORT OTO	SRG117 SRG102 SRG96	URO URO PES	SRG132 SRG74 SRG8	GES ORT GES	SRG176 SRG101 SRG14	VAS VAS VAS	SRG187 SRG151 SRG144
Wednesday	A	OPH GYN GYN	SRG184 SRG158 SRG10	NEU NEU	SRG129 SRG124			OTO CAS ORT	SRG183 SRG156 SRG85	URO URO	SRG152 SRG6	ORT GES ORT	SRG216 SRG176 SRG65	VAS VAS VAS	SRG185 SRG121 SRG49
Thrusday	M	OPH OPH GYN	SRG120 SRG110 SRG46	NEU NEU	SRG115 SRG21			OTO ORT	SRG218 SRG91	GYN GYN GYN	SRG114 SRG70 SRG27	ORT GES ORT	SRG102 SRG88 SRG77	PLS VAS	SRG195 SRG49
Thrusday	A	GYN GYN OPH	SRG192 SRG178 SRG104	NEU NEU	SRG217 SRG124			ORT OTO ORT	SRG106 SRG83 SRG77			ORT GES GES	SRG130 SRG116 SRG20	PLS	SRG105
Friday	M	GYN GYN GYN	SRG119 SRG45 SRG22	MAS NEU	SRG211 SRG99			ORT OTO OTO	SRG130 SRG73 SRG69	GYN PES URO	SRG42 SRG41 SRG5	GES ORT ORT	SRG143 SRG52 SRG15	VAS PLS VAS	SRG160 SRG105 SRG97
Friday	A	OPH OPH GYN	SRG219 SRG197 SRG62	NEU	SRG189			ORT OTO OTO	SRG77 SRG72 SRG63			ORT ORT	SRG201 SRG106	VAS PLS	SRG160 SRG123

Figure 5.8: Solution for algorithm run 23, which improves the implemented MSS

Chapter 6

Conclusions and future work

This work presents a multiobjective MILP model to develop an automated MSS. The aim is: to level the number of patients sent on each day to each hospitalization unit; to concentrate surgeons of the same surgical specialty as much as possible in the same room; to assign surgical specialties to an OR time block when the maximum number of surgeons not individually assigned are available; and to assign OR time to each surgeon or surgical specialty as close as possible to the median time used on the last trimester. The surgeries duration is assumed to be stochastic when incorporated into the model. The required input data have been collected from the medium-sized Portuguese private hospital, containing detailed information on all surgeries performed during 2013 and 2014. The number of required OR time blocks per surgical specialty was not given as input (from the previous decision level, case mix planning) so it is incorporated as a variable into the model. Multiple constraints regarding the structure of the problem, and the special requirements of the hospital under study are taken into account. No other work was found, in the literature, involving the same problem specifications and approach, thus this work provides a valid contribution to the literature in this field. The results obtained are therefore not comparable.

The model does not provide an overall best solution, since different algorithm runs (i.e. different weight values to the four criteria of the objective function) lead to different solutions, and it is up to the surgical suite manager to decide which is the best schedule. This approach enables the hospital surgical suite to be more efficiently managed. Namely it helps to get a better understanding of the trade-offs on the relevant criteria for building a schedule, e.g. how surgeons from the same surgical specialty can be assigned to the same room without compromising the surgical specialty assignment (in terms of the surgeons available on the allocated OR time block), or how demand for the hospitalization units can be leveled without compromising the under and over assignment of OR time to surgeons and surgical specialties. The computational results show that the built-in model generally succeeds in creating appropriate schedules. However, when trying to level the workload sent to each hospitalization unit, while trying to concentrate the surgeons from the same surgical specialty as much as possible in the same room, without considering the deviations to the median value of the OR time used in the last trimester by each surgeon or surgical specialty, the model faces computational difficulties.

The scope of this case study appeal for further research related to the seasonality of the surgical demand, and the opportunity to plan slacks of the MSS. For example, studying the yearly seasonality of the demand for surgical procedures (and the ascending trend) could help to better establish the forecast of the number and type of surgical procedures for each quarter. Using this metric rather than the median OR time used by each surgeon or surgical specialty on the last trimester could improve the accuracy of the MSS. Including planned slacks in the construction of the MSS helps reducing the overtime caused by the variability of the duration of the elective surgeries (Hans et al. 2008). It also helps reducing the overtime caused by arrival of emergency surgeries. However, this was not the aim of the approach.

Moreover, the study undertaken can be broken down into three phases: the processing of the surgical data to be initialized by the MILP model, the implementation and execution of the model, as well as the parameter setting of the terms of the objective function, and the conversion of the model solution into a MSS. The first phase is the one that requires more “human touch”, and so the one that is still not automated, even though it was developed an Excel file with some functions that facilitate this task. The second phase is automated, since it only requires the hospital to have a software license and a person to click on the execution button. The third phase is almost perfectly automated, since it only requires the hospital to have a person who copies the solution obtained from a text file to a provided Excel file, which has a macro to convert the model solution into a MSS. Thus the challenges are to train a person to execute the manual tasks (especially the ones from the first phase), and to persuade the hospital to acquire a software license. Alternatively, a tool can be developed to perform the manual tasks that could not be avoided on this study, and that makes use of a freeware to optimize the model. These further work decisions depend on the feedback of the hospital.

Appendix A

MILP model formulation

A.1 Notation

The notation used in the MILP model is as follows:

- Indices
  - d: days
  - h: hospitalization units
  - k: shifts
  - p: specialties
  - q: classes of the stochastic variables DUR_s^{SRG} and DUR_p^{SPC}
  - r: rooms
  - s: surgeons
- Sets
  - A: set of active days during a week; $|A|$: number of elements in the set A
  - H: set of hospitalization units
  - K: set of shifts; $|K|$: number of elements in the set K
  - P: set of specialties
  - Q_p^{SPC}: sets of classes of the stochastic variables DUR_p^{SPC}
  - Q_s^{SRG}: sets of classes of the stochastic variables DUR_s^{SRG}
  - R: set of rooms
  - S: set of surgeons

- Decision variables
  - $durb_{srdk}$ = the duration (in minutes) of the OR time block allocated to surgeon s in room r on day d and shift k
  - $x_{srdk} = \begin{cases} 1, \text{ if surgeon } s \text{ obtains an OR time block in room } r \text{ on day } d \text{ and shift } k \\ 0, \text{ otherwise} \end{cases}$
  - $y_{prdk} = \begin{cases} 1, \text{ if surgical specialty } p \text{ is assigned to room } r \text{ on day } d \text{ and shift } k \\ 0, \text{ otherwise} \end{cases}$
- Auxiliary variables
  - $\overline{ad_h}$ = the peak weighted absolute deviation of the number of patients sent to the hospitalization unit h over all active days
  - ad_{dh} = the weighted absolute deviation of the number of patients sent to the hospitalization unit h on day d
  - d^{SPC}_{pdhq} = the contribution of allocating an OR time block to surgical specialty p on day d to the absolute deviation from the mean of the number of patients sent to the hospitalization unit h, if the value of the stochastic variable DUR^{SPC}_p belongs to class q
  - d^{SRG}_{sdhq} = the contribution of allocating an OR time block to surgeon s on day d to the absolute deviation from the mean of the number of patients sent to the hospitalization unit h, if the value of the stochastic variable DUR^{SRG}_s belongs to class q
  - $dev^-_{pdk} = \begin{cases} 1, \text{ if the maximum number of available and not individually assigned surgeons that belong to surgical specialty } p \text{ does not occur on day } d \text{ and shift } k\text{, or if it occurs but still the surgical specialty is not assigned to the specific OR time block} \\ 0, \text{ otherwise} \end{cases}$
  - $dev^+_{pdk} = \begin{cases} 1, \text{ if surgical specialty } p \text{ is assigned on day } d \text{ and shift } k \text{ even though the maximum number of available and not individually assigned surgeons of the surgical specialty does not occur on the specific OR time block} \\ 0, \text{ otherwise} \end{cases}$
  - dev^{SPC}_{p-} = the negative deviation of the weekly duration assigned to surgical specialty p to the median value of the weekly time used, in the last trimester, by the surgeons from the same surgical specialty with no individual assignment to OR time blocks
  - dev^{SPC}_{p+} = the positive deviation of the weekly duration assigned to surgical specialty p to the median value of the weekly time used, in the last trimester, by the surgeons from the same surgical specialty with no individual assignment to OR time blocks

- dev_{s-}^{SRG} = the negative deviation of the weekly duration assigned to surgeon s to the median value of the weekly time used by the surgeon in the last trimester
- dev_{s+}^{SRG} = the positive deviation of the weekly duration assigned to surgeon s to the median value of the weekly time used by the surgeon in the last trimester
- $Idev_{pdk}$ = the deviation between the maximum number of available and not individually assigned surgeons that belong to the surgical specialty p and the number of available and not individually assigned surgeons that belong to the surgical specialty p on day d and shift k
- Max_p = the maximum number of available and not individually assigned surgeons that belong to the surgical specialty p
- m_{pdh}^{SPC} = the contribution of allocating an OR time block to surgical specialty p on day d to the mean number of patients sent to the hospitalization unit h
- m_{sdh}^{SRG} = the contribution of allocating an OR time block to surgeon s on day d to the mean number of patients sent to the hospitalization unit h
- $\overline{mean_h}$ = the peak mean number of patients sent to the hospitalization unit h over all active days
- $mean_{dh}$ = the mean number of patients sent to the hospitalization unit h on day d
- n_{pdq}^{SPC} = number of patients that a surgeon of surgical specialty p can operate on day d, if the value of the stochastic variable DUR_p^{SPC} belongs to class q
- n_{sdq}^{SRG} = number of patients that surgeon s can operate on day d, if the value of the stochastic variable DUR_s^{SRG} belongs to class q
- $room_{pr} = \begin{cases} 1, \text{ if at least one surgeon of surgical specialty } p \text{ obtains an OR time block} \\ \quad \text{in room } r \\ 0, \text{ otherwise} \end{cases}$
- u_{srdk} = (integer) number of thirty-minute time blocks of the OR time block allocated to surgeon s in room r on day d and shift k
- $z_{rdk} = \begin{cases} 1, \text{ if an OR time block in room } r \text{ on day } d \text{ and shift } k \text{ is assigned to at least} \\ \quad \text{one surgeon} \\ 0, \text{ otherwise} \end{cases}$
- $z_{sd}^{day} = \begin{cases} 1, \text{ if surgeon } s \text{ obtains at least one OR time block on day } d \\ 0, \text{ otherwise} \end{cases}$
- $z_s^{week} = \begin{cases} 1, \text{ if surgeon } s \text{ obtains at least one OR time block during the week} \\ 0, \text{ otherwise} \end{cases}$

- Parameters
  - $a_{sdk}: \begin{cases} 1, \text{ if surgeon } s \text{ is available on day } d \text{ and shift } k \\ 0, \text{ otherwise} \end{cases}$
  - $b_{sp}: \begin{cases} 1, \text{ if surgeon } s \text{ belongs to surgical specialty } p \\ 0, \text{ otherwise} \end{cases}$
  - $capacity_{rdk}$: the total capacity (in minutes) of room r on day d and shift k, which is defined as a multiple of thirty-minute time blocks
  - DUR_p^{SPC} = stochastic variables representing the duration of a surgery (in minutes) performed by a surgeon from the surgical specialty p, including the induction and waking time, and the cleaning procedures
  - dur_{pq}^{SPC}: the midpoint of class q, with $q \in Q_p^{SPC}$, of the stochastic variable DUR_p^{SPC}
  - DUR_s^{SRG} = stochastic variables representing the duration of a surgery (in minutes) performed by surgeon s, including the induction and waking time, and the cleaning procedures
  - dur_{sq}^{SRG}: the midpoint of class q, with $q \in Q_s^{SRG}$, of the stochastic variable DUR_s^{SRG}
  - max_s^{day}: the maximum duration (as a multiple of a thirty-minute time blocks) that can be daily assigned to surgeon s
  - max_s^{week}: the maximum duration (as a multiple of a thirty-minute time blocks) that can be weekly assigned to surgeon s
  - $median_s$: the median value of the weekly time (in minutes) used by surgeon s in the last trimester
  - min_s^{day}: the minimum duration (as a multiple of a thirty-minute time blocks) that can be daily assigned to surgeon s
  - min_s^{week}: the minimum duration (as a multiple of a thirty-minute time blocks) that can be weekly assigned to surgeon s
  - $pdur_{pq}^{SPC}$: the probability of the duration of a surgery, performed by a surgeon of surgical specialty p, belongs to class q, with $q \in Q_p^{SPC}$, of the stochastic variable DUR_p^{SPC}
  - $pdur_{sq}^{SRG}$: the probability of the duration of a surgery, performed by surgeon s, belongs to class q, with $q \in Q_s^{SRG}$, of the stochastic variable DUR_s^{SRG}
  - phu_{ph}^{SPC}: the probability for a patient to be sent to the hospitalization unit h, after being operated by a surgeon of surgical specialty p

- phu_{sh}^{SRG}: the probability for a patient to be sent to the hospitalization unit h, after being operated by surgeon s
- Wad_h: the relative importance of leveling the weighted absolute deviation from the mean of the number of patients sent to the hospitalization unit h
- $Wdev_p$: the relative importance of allocating the surgical specialty p on the day and shift in which the highest number of not individually assigned surgeons, that belong to the same surgical specialty, is available
- $Wdev_{p-}^{SPC}$: the relative importance of diverging negatively the weekly duration assigned to surgical specialty p to the median value of the weekly time used, in the last trimester, by the surgeons from the same surgical specialty with no individual assignment to OR time blocks
- $Wdev_{p+}^{SPC}$: the relative importance of diverging positively the weekly duration assigned to surgical specialty p to the median value of the weekly time used, in the last trimester, by the surgeons from the same surgical specialty with no individual assignment to OR time blocks
- $Wdev_{s-}^{SRG}$: the relative importance of diverging negatively the weekly duration assigned to surgeon s to the median value of the weekly time used in the last trimester
- $Wdev_{s+}^{SRG}$: the relative importance of diverging positively the weekly duration assigned to surgeon s to the median value of the weekly time used in the last trimester
- $Wmean_h$: the relative importance of leveling the mean number of patients sent to the hospitalization unit h
- $Wroom_p$: the relative importance of concentrating surgeons that belong to surgical specialty p (as much as possible) in the same room

A.2 MILP model formulation

The objective function formulation divided in the four optimization criteria is firstly presented. Then the problem constraints are stated (in the same order as in chapter 3).

Minimize

$$\underbrace{\sum_{h \in H} Wmean_h \cdot \overline{mean_h} + \sum_{h \in H} Wad_h \cdot \overline{ad_h}}_{\text{1st optimization criterion}} + \underbrace{\sum_{p \in P} Wroom_p \cdot \sum_{r \in R} room_{pr}}_{\text{2nd optimization criterion}} + \underbrace{\sum_{p \in P} Wdev_p \cdot \sum_{d \in A} \sum_{k \in K} dev_{pdk}^{+}}_{\text{3rd optimization criterion}}$$

$$+ \underbrace{\sum_{s \in S} Wdev_{s-}^{SRG} \cdot dev_{s-}^{SRG} + \sum_{s \in S} Wdev_{s+}^{SRG} \cdot dev_{s+}^{SRG} + \sum_{p \in P} Wdev_{p-}^{SPC} \cdot dev_{p-}^{SPC} + \sum_{p \in P} Wdev_{p+}^{SPC} \cdot dev_{p+}^{SPC}}_{\text{4th optimization criterion}}$$

Subject to:

$$\sum_{r\in R} x_{srdk} \le a_{sdk},\ \forall s\in S,\ d\in A,\ k\in K \tag{3.1}$$

$$\sum_{s\in S} x_{srdk} \le 3\cdot z_{rdk},\ \forall r\in R,\ d\in A,\ k\in K \tag{3.2}$$

$$\sum_{s\in S} durb_{srdk} = capacity_{rdk}\cdot z_{rdk},\ \forall r\in R,\ d\in A,\ k\in K \tag{3.3}$$

$$z_{rdk} + \sum_{p\in P} y_{prdk} \le 1,\ \forall r\in R,\ d\in A,\ k\in K \tag{3.4}$$

$$durb_{srdk} \le capacity_{rdk}\cdot x_{srdk},\ \forall s\in S,\ r\in R,\ d\in A,\ k\in K \tag{3.5}$$

$$durb_{srdk} = 30\cdot u_{srdk},\ \forall s\in S,\ r\in R,\ d\in A,\ k\in K \tag{3.6}$$

$$min_s^{day}\cdot z_{sd}^{day} \le \sum_{r\in R}\sum_{k\in K} durb_{srdk} \le max_s^{day}\cdot z_{sd}^{day},\ \forall s\in S,\ d\in A \tag{3.7}$$

$$min_s^{week}\cdot z_s^{week} \le \sum_{r\in R}\sum_{d\in A}\sum_{k\in K} durb_{srdk} \le max_s^{week}\cdot z_s^{week},\ \forall s\in S \tag{3.8}$$

$$\sum_{r\in R}\sum_{k\in K} x_{srdk} \le |K|\cdot z_{sd}^{day},\ \forall s\in S,\ d\in A \tag{3.9}$$

$$\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} x_{srdk} \le |A|\cdot|K|\cdot z_s^{week},\ \forall s\in S \tag{3.10}$$

$$\sum_{s\in S}\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} durb_{srdk} \ge 0.75\cdot\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} capacity_{rdk} \tag{3.11}$$

$$\sum_{r\in R}\sum_{d\in A}\sum_{k\in K}\Big(\sum_{s\in S} durb_{srdk} + capacity_{rdk}\cdot\sum_{p\in P} y_{prdk}\Big) \ge 0.8\cdot\sum_{r\in R}\sum_{d\in A}\sum_{k\in K} capacity_{rdk} \tag{3.12}$$

$$n_{sdq}^{SRG} = \sum_{r\in R}\sum_{k\in K}\frac{durb_{srdk}}{dur_{sq}^{SRG}},\ \forall s\in S,\ d\in A,\ q\in Q_s^{SRG} \tag{3.13}$$

$$n_{pdq}^{SPC} = \sum_{r\in R}\sum_{k\in K}\frac{capacity_{rdk}\cdot y_{prdk}}{dur_{pq}^{SPC}},\ \forall p\in P,\ d\in A,\ q\in Q_p^{SPC} \tag{3.14}$$

$$m_{sdh}^{SRG} = \sum_{q\in Q_s^{SRG}} pdur_{sq}^{SRG}\cdot phu_{sh}^{SRG}\cdot n_{sdq}^{SRG},\ \forall s\in S,\ d\in A, h\in H \tag{3.15}$$

$$m_{pdh}^{SPC} = \sum_{q\in Q_p^{SPC}} pdur_{pq}^{SPC}\cdot phu_{ph}^{SPC}\cdot n_{pdq}^{SPC},\ \forall p\in P,\ d\in A,\ h\in H \tag{3.16}$$

$$d_{sdhq}^{SRG} \ge phu_{sh}^{SRG}\cdot n_{sdq}^{SRG} - m_{sdh}^{SRG}, \forall s\in S,\ d\in A,\ h\in H,\ q\in Q_s^{SRG} \tag{3.17}$$

$$d_{sdhq}^{SRG} \ge m_{sdh}^{SRG} - phu_{sh}^{SRG}\cdot n_{sdq}^{SRG}, \forall s\in S,\ d\in A,\ h\in H,\ q\in Q_s^{SRG} \tag{3.18}$$

$$d_{pdhq}^{SPC} \ge phu_{ph}^{SPC}\cdot n_{pdq}^{SPC} - m_{pdh}^{SPC}, \forall p\in P,\ d\in A,\ h\in H,\ q\in Q_p^{SPC} \tag{3.19}$$

$$d_{pdhq}^{SPC} \ge m_{pdh}^{SPC} - phu_{ph}^{SPC}\cdot n_{pdq}^{SPC}, \forall p\in P,\ d\in A,\ h\in H,\ q\in Q_p^{SPC} \tag{3.20}$$

$$mean_{dh} = \sum_{s \in S} m_{sdh}^{SRG} + \sum_{p \in P} m_{pdh}^{SPC}, \forall d \in A, h \in H \tag{3.21}$$

$$ad_{dh} = \sum_{s \in S} \sum_{q \in Q_s^{SRG}} pdur_{sq}^{SRG} \cdot d_{sdhq}^{SRG} + \sum_{p \in P} \sum_{q \in Q_p^{SPC}} pdur_{pq}^{SPC} \cdot d_{pdhq}^{SPC}, \forall d \in A, h \in H \tag{3.22}$$

$$mean_{dh} \leq \overline{mean_h}, \forall d \in A, h \in H \tag{3.23}$$

$$ad_{dh} \leq \overline{ad_h}, \forall d \in A, h \in H \tag{3.24}$$

$$\sum_{d \in A} \sum_{k \in K} \left(\sum_{s \in S} x_{srdk} \cdot b_{sp} + y_{prdk} \right) \leq |A| \cdot |K| \cdot \left(\sum_{s \in S} b_{sp} + 1 \right) \cdot room_{pr}, \forall p \in P, r \in R \tag{3.26}$$

$$Max_p - Idev_{pdk} = \sum_{s \in S} b_{sp} \cdot a_{sdk} \cdot (1 - z_s^{week}), \forall p \in P, d \in A, k \in K \tag{3.29}$$

$$Idev_{pdk} \leq \sum_{s \in S} b_{sp} \cdot dev_{pdk}^{-}, \forall p \in P, d \in A, k \in K \tag{3.30}$$

$$\sum_{r \in R} y_{prdk} + dev_{pdk}^{-} - dev_{pdk}^{+} = 1, \forall p \in P, d \in A, k \in K \tag{3.31}$$

$$\sum_{r \in R} \sum_{d \in A} \sum_{k \in K} y_{prdk} \leq 1, \forall p \in P \tag{3.33}$$

$$\sum_{r \in R} \sum_{d \in A} \sum_{k \in K} y_{prdk} \leq Max_p, \forall p \in P \tag{3.34}$$

$$\sum_{r \in R} \sum_{d \in A} \sum_{k \in K} durb_{srdk} + dev_{s-}^{SRG} - dev_{s+}^{SRG} = median_s, \forall s \in S \tag{3.35}$$

$$\sum_{r \in R} \sum_{d \in A} \sum_{k \in K} capacity_{rdk} \cdot y_{prdk} + dev_{p-}^{SPC} - dev_{p+}^{SPC} = \sum_{s \in S} b_{sp} \cdot median_s \cdot (1 - z_s^{week}), \forall p \in P \tag{3.36}$$

$$durb_{srdk} \geq 0 \text{ and integer}, \forall s \in S, r \in R, d \in A, k \in K \tag{3.38}$$

$$x_{srdk} \in \{0, 1\}, \forall s \in S, r \in R, d \in A, k \in K \tag{3.39}$$

$$y_{prdk} \in \{0, 1\}, \forall p \in P, r \in R, d \in A, k \in K \tag{3.40}$$

$$dev_{pdk}^{-}, dev_{pdk}^{+} \in \{0, 1\}, \forall p \in P, d \in A, k \in K \tag{3.41}$$

$$room_{pr} \in \{0, 1\}, \forall p \in P, r \in R \tag{3.42}$$

$$z_{rdk} \in \{0, 1\}, \forall r \in R, d \in A, k \in K \tag{3.43}$$

$$z_{sd}^{day} \in \{0, 1\}, \forall s \in S, d \in A \tag{3.44}$$

$$z_s^{week} \in \{0, 1\}, \forall s \in S \tag{3.45}$$

$$dev_{p-}^{SPC},\, dev_{p+}^{SPC} \geq 0 \text{ and integer, } \forall p \in P \tag{3.46}$$

$$dev_{s-}^{SRG},\, dev_{s+}^{SRG} \geq 0 \text{ and integer, } \forall s \in S \tag{3.47}$$

$$Idev_{pdk} \geq 0 \text{ and integer, } \forall p \in P \tag{3.48}$$

$$Max_p \geq 0 \text{ and integer, } \forall p \in P \tag{3.49}$$

$$u_{srdk} \geq 0 \text{ and integer, } \forall s \in S,\, r \in R,\, d \in A,\, k \in K \tag{3.50}$$

$$\overline{ad_h},\, \overline{mean_h} \geq 0,\, \forall h \in H \tag{3.51}$$

$$ad_{dh},\, mean_{dh} \geq 0,\, d \in A,\, h \in H \tag{3.52}$$

$$d_{pdhq}^{SPC} \geq 0,\, \forall p \in P,\, d \in A,\, h \in H,\, q \in Q_p^{SPC} \tag{3.53}$$

$$d_{sdhq}^{SRG} \geq 0,\, \forall s \in S,\, d \in A,\, h \in H,\, q \in Q_s^{SRG} \tag{3.54}$$

$$m_{pdh}^{SPC} \geq 0,\, \forall p \in P,\, d \in A,\, h \in H \tag{3.55}$$

$$m_{sdh}^{SRG} \geq 0,\, \forall s \in S,\, d \in A,\, h \in H \tag{3.56}$$

$$n_{pdq}^{SPC} \geq 0,\, \forall p \in P,\, d \in A,\, q \in Q_p^{SPC} \tag{3.57}$$

$$n_{sdq}^{SRG} \geq 0,\, \forall s \in S,\, d \in A,\, q \in Q_s^{SRG} \tag{3.58}$$

References

Beliën, J. & Demeulemeester, E. (2007), 'Building cyclic master surgery schedules with leveled resulting bed occupancy', *European Journal of Operational Research* **176**(2), 1185–1204.

Beliën, J., Demeulemeester, E. & Cardoen, B. (2006), 'Visualizing the demand for various resources as a function of the master surgery schedule: A case study', *Journal of Medical Systems* **30**(5), 343–350.

Beliën, J., Demeulemeester, E. & Cardoen, B. (2009), 'A decision support system for cyclic master surgery scheduling with multiple objectives', *Journal of Scheduling* **12**(2), 147–161.

Blake, J. T. & Carter, M. W. (2002), 'A goal programming approach to strategic resource allocation in acute care hospitals', *European Journal of Operational Research* **140**(3), 541–561.

Blake, J. T., Dexter, F. & Donald, J. (2002), 'Operating room managers' use of integer programming for assigning block time to surgical groups: A case study', *Anesthesia & Analgesia* **94**(1), 143–148.

Blake, J. T. & Donald, J. (2002), 'Mount sinai hospital uses integer programming to allocate operating room time', *Interfaces* **32**(2), 63–73.

Cappanera, P., Visintin, F. & Banditori, C. (2014), 'Comparing resource balancing criteria in master surgical scheduling: A combined optimisation-simulation approach', *International Journal of Production Economics* **158**, 179–196.

Cardoen, B., Demeulemeester, E. & Beliën, J. (2009), 'Sequencing surgical cases in a day-care environment: An exact branch-and-price approach', *Computers & Operations Research* **36**(9), 2660–2669.

Cardoen, B., Demeulemeester, E. & Beliën, J. (2010), 'Operating room planning and scheduling: A literature review', *European Journal of Operational Research* **201**(3), 921–932.

Colombani, Y. & Heipcke, S. (2002), 'Mosel: An extensible environment for modeling and programming solutions', `http://web.emn.fr/x-info/cpaior/Proceedings/CPAIOR-20.pdf`. Accessed August 4, 2016.

Company, A. B. (2001), *Surgical services reform: Executive briefing for clinical leaders*, Clinical Advisory Board, Advisory Board Company.

Fei, H., Meskens, N. & Chu, C. (2010), 'A planning and scheduling problem for an operating theatre using an open scheduling strategy', *Computers & Industrial Engineering* **58**(2), 221–230.

Fügener, A., Hans, E. W., Kolisch, R., Kortbeek, N. & Vanberkel, P. T. (2014), 'Master surgery scheduling with consideration of multiple downstream units', *European Journal of Operational Research* **239**(1), 227–236.

Gomes, C. (2014), Optimizing operating theater planning: A data mining and optimization approach, Master's thesis, Universidade do Porto.

Guerriero, F. & Guido, R. (2011), 'Operational research in the management of the operating theatre: a survey', *Health Care Management Science* **14**(1), 89–114.

Hans, E., Wullink, G., Van Houdenhoven, M. & Kazemier, G. (2008), 'Robust surgery loading', *European Journal of Operational Research* **185**(3), 1038–1050.

Hosseini, N. & Taaffe, K. M. (2015), 'Allocating operating room block time using historical caseload variability', *Health Care Management Science* **18**(4), 419–430.

IBM Decision Optimization on Cloud (2016), `https://dropsolve-oaas.docloud.ibmcloud.com/`. Accessed August 4, 2016.

Kuo, P. C., Schroeder, R. A., Mahaffey, S. & Bollinger, R. R. (2003), 'Optimization of operating room allocation using linear programming techniques', *Journal of the American College of Surgeons* **197**(6), 889–895.

Mannino, C., Nilssen, E. J. & Nordlander, T. E. (2012), 'A pattern based, robust approach to cyclic master surgery scheduling', *Journal of Scheduling* **15**(5), 553–563.

Marchesi, J. F. & Pacheco, M. A. C. (2016), 'A genetic algorithm approach for the master surgical schedule problem', *2016 IEEE Conference on Evolving and Adaptive Intelligent Systems* pp. 17–21.

Marques, I., Captivo, M. E. & Pato, M. V. (2012), 'An integer programming approach to elective surgery scheduling', *OR Spectrum* **34**(2), 407–427.

Marques, I., Captivo, M. E. & Pato, M. V. (2014), 'Scheduling elective surgeries in a portuguese hospital using a genetic heuristic', *Operations Research for Health Care* **3**(2), 59–72.

Testi, A., Tanfani, E. & Torre, G. (2007), 'A three-phase approach for operating theatre schedules', *Health Care Management Science* **10**(2), 163–172.

The R Project for Statistical Computing (2015), `https://www.r-project.org/`. Accessed August 4, 2016.

van Essen, J. T., Bosch, J. M., Hans, E. W., van Houdenhoven, M. & Hurink, J. L. (2014), 'Reducing the number of required beds by rearranging the or-schedule', *OR Spectrum* **36**(3), 585–605.

Van Oostrum, J. M., Bredenhoff, E. & Hans, E. W. (2010), 'Suitability and managerial implications of a master surgical scheduling approach', *Annals of Operations Research* **178**(1), 91–104.

van Oostrum, J. M., Van Houdenhoven, M., Hurink, J. L., Hans, E. W., Wullink, G. & Kazemier, G. (2008), 'A master surgical scheduling approach for cyclic scheduling in operating room departments', *OR Spectrum* **30**(2), 355–374.

Vanberkel, P. T., Boucherie, R. J., Hans, E. W., Hurink, J. L., Van Lent, W. A. & Van Harten, W. H. (2011), 'An exact approach for relating recovering surgical patient workload to the master surgical schedule', *Journal of the Operational Research Society* **62**(10), 1851–1860.

Vissers, J., Adan, I. J. & Bekkers, J. A. (2005), 'Patient mix optimization in tactical cardiothoracic surgery planning: A case study', *IMA Journal of Management Mathematics* **16**(3), 281–304.